From classical electrodynamics, a rotating electrically charged body creates a magnetic dipole with magnetic poles of equal magnitude but opposite polarity. This analogy holds as an electron indeed behaves like a tiny bar magnet. In the classical model, the electron (Bohr atom model) is moving along a closed path. A revolving electron is equivalent to a current loop (Figure \(\PageIndex{1a}\)).

![Current-carrying loop and Hydrogen atom](image)

Figure \(\PageIndex{1}\): The rotating electron (really intrinsic spin) of the electron induces a different magnet. Image used with permission (CC BY-SA OpenStax).

The current arising from the orbiting of the electron around the nucleus is

\[ I = \frac{e}{T} \]

where \(T\) is the orbital period or revolution. We can define the velocity then as

\[ v = \frac{2\pi r}{T} \]

where \(r\) is the radius of orbit. The "orbital" magnetic (dipole) moment is then

\[ \vec{\mu}_L = \frac{\vec{\mu}e}{T} (\pi r^2) \tag{eq1} \]

and the angular momentum associated with that orbital motion is

\[
\begin{align*}
L &= m_e v_r \\
&= \frac{m_e 2\pi r^2}{T} \tag{eq2}
\end{align*}
\]

and \(m_e\) is the electron rest mass.

Next we combine Equation \ref{eq1} and \ref{eq2} to get how the orbital angular momentum induced a magnetic moment (in vector form)

\[ \vec{\mu}_L = -\frac{e}{2m_e} \vec{L} \tag{classical} \]

For a more detailed derivation of Equation \ref{classical}, look here. We can glean two important feature from Equation \ref{classical}:

1. If the electron has zero orbital angular momentum, it will have a zero-amplitude orbital magnetic moment.
2. The orientation of the orbital angular moment is parallel to the angular momentum.

The Quantum Orbital Magnetic Dipole Moment

Equation \ref{classical} is applicable to classical systems and since \(\vec{L}\) can have any values (amplitude and
orientation), so can the magnetic moment. That is a different story in the quantum world. Where the orbital angular momentum is quantized and hence so is \(\mu_L\). The amplitude of is \(\langle \vec{L} \rangle\) represented by the \(\hat{L}^2\) operator and the projection of \(\langle \vec{L} \rangle\) on the z-axis is represented by \(\langle \hat{L}_z \rangle\) operator.

\[
\langle \hat{L}^2 \rangle \langle \psi \rangle = \ell(\ell+1) \hbar^2 \langle \psi \rangle
\]

and

\[
\langle \hat{L}_z \rangle \langle \psi \rangle = m_l \hbar \langle \psi \rangle
\]

So we obtain the eigenvalues \(\mu_{L_z}\) of z-component of the magnetic moment

\[
\mu_{L_z} = -\dfrac{e}{2m_e} \hbar m_l \tag{eq5}
\]

It is usual to express the magnetic moment (Equation \ref{eq5}) in terms the Bohr magneton \(\mu_B\)

\[
\mu_{L_z} = -\mu_B m_l
\]

where \(\mu_B\) is the Bohr Magneton:

\[
\mu_B = \dfrac{e\hbar}{2m_e}
\]

The Bohr magneton is a physical constant and the natural unit for expressing the magnetic moment of an electron caused by either its orbital angular momentum and (and spin as discussed below) and is numerically

\[
\begin{align}
\mu_B &= 9.273 \times 10^{-24} \text{ J/K}, J/K \\
&= 5.656 \times 10^{-5} \text{ eV/T}, eV/T \\
&= 1.4 \times 10^{10} \text{ Hz/T}, (T = \text{Tesla})
\end{align}
\]

---

### Coupling Orbital Magnetic Dipole Moment to an External Magnetic Field

When a magnetic field is applied to the electron, then the energies of the eigenstates will depends on the magnitude \(B\) of the applied magnetic field \(\langle \vec{B} \rangle\) via

\[
\begin{align}
E_B &= -\mu_{L_z} B \\
&= \mu_B m_l B
\end{align}
\]

and the total energy of that state is

\[
\begin{align}
E &= E_0 + E_B \\
&= E_0 + \mu_B m_l B
\end{align}
\]

Hence, the energy levels having a angular momentum with quantum number \(\ell\) are split into \((2\ell + 1)\) new levels (Figure \PageIndex(2)\):
Figure \(\PageIndex{2}\): Splitting of spin-1/2 state into two in an external magnetic field

This effect is the "ordinary" or "normal" Zeeman Effect.

Coupling Spin Magnetic Dipole Moment to an External Magnetic Field

The hydrogen ground state \((m_l = 0)\) is uninfluenced in Figure \(\PageIndex{1}\). However, we know that a H atom is paramagnetic; this is because of electron spin. Now we consider the spin in classical mechanics as rotating around the axis electron. We also find here

\[
\mu_{S_z} = -g_S \cdot \mu_B \cdot m_s
\]

\[
E_B = g_S \cdot \mu_B \cdot m_s \cdot B
\]

The so-called gyromagnetic factor \((g_S)\) is obtained from the relativistic Dirac equation and experiments quantify it at

\[
g_S = 2.00231930438(6)
\]

The value

\[
g_S \approx 28 \text{ GHz/T}
\]
shows us which energy state is higher according to electron spin interaction with magnetic field. Since now it's possible to produce magnetic fields with the strength of a few Teslas, we expect to detect transitions in GHz region (microwaves) when applying such fields. That's why Electron-Spin-Resonance (ESR) Spectroscopy is involved with microwaves. When applying NMR method we have a deal with MHz region. We will discuss it in more detail later.