A group that contains a large number of symmetry elements may often be constructed from simpler groups. This is probably best illustrated using an example. Consider the point groups \( \langle C_2 \rangle \) and \( \langle C_S \rangle \). \( \langle C_2 \rangle \) contains the elements \( \lbrace E \rbrace \) and \( \lbrace C_2 \rbrace \), and has order 2, while \( \langle C_S \rangle \) contains \( \lbrace E \rbrace \) and \( \sigma \) and also has order \( \langle 2 \rangle \). We can use these two groups to construct the group \( \langle C_{2v} \rangle \) by applying the symmetry operations of \( \langle C_2 \rangle \) and \( \langle C_S \rangle \) in sequence.

\[
\begin{array}{lllll}
C_2 \text{ operation} & E & E & C_2 & C_2 \\
C_S \text{ operation} & E & \sigma(xz) & E & \sigma(xz) \\
\text{Result} & E & \sigma_v(xz) & C_2 & \sigma_v'(yz)
\end{array}
\tag{6.1}
\]

Notice that \( \langle C_{2v} \rangle \) has order \( \langle 4 \rangle \), which is the product of the orders of the two lower-order groups. \( \langle C_{2v} \rangle \) may be described as a direct product group of \( \langle C_2 \rangle \) and \( \langle C_S \rangle \). The origin of this name should become obvious when we review the properties of matrices.

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