Now we will investigate what happens when we apply two symmetry operations in sequence. As an example, consider the \( \text{NH}_3 \) molecule, which belongs to the \( \text{C}_{3v} \) point group. Consider what happens if we apply a \( \text{C}_3 \) rotation followed by a \( \sigma_v \) reflection. We write this combined operation \( \sigma_v \text{C}_3 \) (when written, symmetry operations operate on the thing directly to their right, just as operators do in quantum mechanics – we therefore have to work backwards from right to left from the notation to get the correct order in which the operators are applied). As we shall soon see, the order in which the operations are applied is important.

The combined operation \( \sigma_v \text{C}_3 \) is equivalent to \( \sigma_v'' \), which is also a symmetry operation of the \( \text{C}_{3v} \) point group. Now let’s see what happens if we apply the operators in the reverse order i.e. \( \text{C}_3 \sigma_v \) (\( \sigma_v \) followed by \( \text{C}_3 \)).

Again, the combined operation \( \text{C}_3 \sigma_v \) is equivalent to another operation of the point group, this time \( \sigma_v' \).

There are two important points that are illustrated by this example:

1. The order in which two operations are applied is important. For two symmetry operations \( (A) \) and \( (B) \), \( (AB) \) is not necessarily the same as \( (BA) \), i.e. symmetry operations do not in general commute. In some groups the symmetry elements do commute; such groups are said to be Abelian.

2. If two operations from the same point group are applied in sequence, the result will be equivalent to another operation from the point group. Symmetry operations that are related to each other by other symmetry operations of the group are said to belong to the same class. In \( \text{NH}_3 \), the three mirror planes \( \sigma_v \), \( \sigma_v' \) and \( \sigma_v'' \) belong to the same class (related to each other through a \( \text{C}_3 \) rotation), as do the rotations \( \text{C}_3^+ \) and \( \text{C}_3^- \) (anticlockwise and clockwise rotations about the principal axis, related to each other by a vertical mirror plane).

The effects of applying two symmetry operations in sequence within a given point group are summarized in group multiplication tables. As an example, the complete group multiplication table for \( \text{C}_{3v} \) using the symmetry operations as defined in the figures above is shown below. The operations written along the first row of the table are carried out first, followed by those written in the first column (note that the table would change if we chose to name \( \sigma_v \), \( \sigma_v' \) and \( \sigma_v'' \) in some different order).

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**Contributors and Attributions**

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