It is often required to know thermodynamic functions (such as enthalpy) at temperatures other than those available from tabulated data. Fortunately, the conversion to other temperatures is not difficult.

At constant pressure

\[
\frac{\Delta H}{\Delta T} = C_p
\]

And so for a temperature change from \((T_1)\) to \((T_2)\)

\[
\Delta H = \int_{T_2}^{T_2} C_p\,dT \label{EQ1}
\]

Equation \ref{EQ1} is often referred to as Kirchhoff's Law. If \((C_p)\) is independent of temperature, then

\[
\Delta H = C_p \Delta T \label{intH}
\]

If the temperature dependence of the heat capacity is known, it can be incorporated into the integral in Equation \ref{EQ1}. A common empirical model used to fit heat capacities over broad temperature ranges is

\[
C_p(T) = a + bT + \frac{c}{T^2} \label{EQ15}
\]

After combining Equations \ref{EQ15} and \ref{EQ1}, the enthalpy change for the temperature change can be found obtained by a simple integration

\[
\Delta H = \int_{T_1}^{T_2} \left(a + bT + \frac{c}{T^2}\right) dT \label{EQ2}
\]

Solving the definite integral yields

\[
\begin{align}
\Delta H &= \left[aT + \frac{b}{2} T^2 - \frac{c}{T}\right]_{T_1}^{T_2} \\
&= a(T_2-T_1) + \frac{b}{2}(T_2^2-T_1^2) - c \left( \frac{1}{T_2} - \frac{1}{T_1} \right) \label{ineq}
\end{align}
\]

This expression can then be used with experimentally determined values of \((a)\), \((b)\), and \((c)\), some of which are shown in the following table.

<table>
<thead>
<tr>
<th>Substance</th>
<th>(a) (J mol(^{-1}) K(^{-1}))</th>
<th>(b) (J mol(^{-1}) K(^{-2}))</th>
<th>(c) (J mol(^{-1}) K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(gr)</td>
<td>16.86</td>
<td>4.77 \times 10^{-3}</td>
<td>-8.54 \times 10^5</td>
</tr>
<tr>
<td>CO(_2)(g)</td>
<td>44.22</td>
<td>8.79 \times 10^{-3}</td>
<td>-8.62 \times 10^5</td>
</tr>
<tr>
<td>H(_2)O(l)</td>
<td>75.29</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>N(_2)(g)</td>
<td>28.58</td>
<td>3.77 \times 10^{-3}</td>
<td>-5.0 \times 10^4</td>
</tr>
<tr>
<td>Pb(s)</td>
<td>22.13</td>
<td>1.172 \times 10^{-2}</td>
<td>9.6 \times 10^4</td>
</tr>
</tbody>
</table>
Example \(\PageIndex{1}\): Heating Lead

What is the molar enthalpy change for a temperature increase from 273 K to 353 K for Pb(s)?

Solution:

The enthalpy change is given by Equation \ref{EQ1} with a temperature dependence \(C_p\) given by Equation \ref{EQ1} using the parameters in Table \(\PageIndex{1}\). This results in the integral form (Equation \ref{ineq}):

\[
\Delta H = a(T_2-T_1) + \dfrac{b}{2}(T_2^2-T_1^2) - c \left( \dfrac{1}{T_2} - \dfrac{1}{T_1} \right)
\]

when substituted with the relevant parameters of Pb(s) from Table \(\PageIndex{1}\).

\[
\begin{align*}
\Delta H &= (22.14 \dfrac{J}{mol\,K} (353\,K - 273\,K) \\
&\quad + \dfrac{1.172 \times 10^{-2} \dfrac{J}{mol\,K^2}}{2} (353^2 - 273^2) \\
&\quad - 9.6 \times 10^4 \dfrac{J\,K}{mol} \left( \dfrac{1}{353\,K} - \dfrac{1}{273\,K} \right)
\end{align*}
\]

\[
\Delta H = 1770.4 \dfrac{J}{mol} + 295.5 \dfrac{J}{mol} + 470.5 \dfrac{J}{mol} = 2534.4 \dfrac{J}{mol}
\]

For chemical reactions, the reaction enthalpy at differing temperatures can be calculated from

\[
\Delta H_{\text{rxn}}(T_2) = \Delta H_{\text{rxn}}(T_1) + \int_{T_1}^{T_2} \Delta C_p \Delta T
\]

Example \(\PageIndex{2}\): Enthalpy of Formation

The enthalpy of formation of \(\text{NH}_3(g)\) is -46.11 kJ/mol at 25°C. Calculate the enthalpy of formation at 100°C.

Solution:

\[
\begin{align*}
\text{N}_2(g) + 3 \text{H}_2(g) \rightleftharpoons 2 \text{NH}_3(g)\\
\Delta H (298\,K) &= -46.11 \text{ kJ/mol} \\
\end{align*}
\]

with \(\Delta H (298\,K) = -46.11\, \text{kJ/mol}\)

<table>
<thead>
<tr>
<th>Compound</th>
<th>(C_p) (J mol(^{-1}) K(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{N}_2(g))</td>
<td>29.12</td>
</tr>
<tr>
<td>(\text{H}_2(g))</td>
<td>28.82</td>
</tr>
<tr>
<td>(\text{NH}_3(g))</td>
<td>35.06</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\Delta H (373\,K) &= \Delta H (298\,K) + \Delta C_p \Delta T \\
&= -46110 + \dfrac{2 \times 29.12 \dfrac{J}{mol\,K} \text{right} - 3 \times 28.82 \dfrac{J}{mol\,K} \text{right}}{373\,K - 298\,K} \\
&= -49.5 \dfrac{kJ}{mol}
\end{align*}
\]