Learning Objectives

• To relate the amount of gas consumed or released in a chemical reaction to the stoichiometry of the reaction.
• To understand how the ideal gas equation and the stoichiometry of a reaction can be used to calculate the volume of gas produced or consumed in a reaction.

With the ideal gas law, we can use the relationship between the amounts of gases (in moles) and their volumes (in liters) to calculate the stoichiometry of reactions involving gases, if the pressure and temperature are known. This is important for several reasons. Many reactions that are carried out in the laboratory involve the formation or reaction of a gas, so chemists must be able to quantitatively treat gaseous products and reactants as readily as they quantitatively treat solids or solutions. Furthermore, many, if not most, industrially important reactions are carried out in the gas phase for practical reasons. Gases mix readily, are easily heated or cooled, and can be transferred from one place to another in a manufacturing facility via simple pumps and plumbing.

Gas Densities and Molar Mass

The ideal-gas equation can be manipulated to solve a variety of different types of problems. For example, the density, \( \rho \), of a gas, depends on the number of gas molecules in a constant volume. To determine this value, we rearrange the ideal gas equation to

\[
\frac{n}{V} = \frac{P}{RT} \quad \text{(10.5.1)}
\]

Density of a gas is generally expressed in g/L (mass over volume). Multiplication of the left and right sides of Equation (10.5.1) by the molar mass in g/mol (\( M \)) of the gas gives

\[
\rho = \frac{g}{L} = \frac{PM}{RT} \quad \text{(10.5.2)}
\]

This allows us to determine the density of a gas when we know the molar mass, or vice versa.

Example

What is the density of nitrogen gas (\( \text{N}_2 \)) at 248.0 Torr and 18º C?

SOLUTION

Step 1: Write down your given information

- \( P = 248.0 \text{ Torr} \)
- \( V = ? \)
- \( n = ? \)
- \( R = 0.0820574 \text{ L} \cdot \text{atm} \cdot \text{mol}^{-1} \cdot \text{K}^{-1} \)
- \( T = 18º \text{ C} \)

Step 2: Convert as necessary.
Step 3: This one is tricky. We need to manipulate the Ideal Gas Equation to incorporate density into the equation.

Write down all known equations:

\[ PV = nRT \]
\[ \rho = \frac{m}{V} \]

where \( \rho \) is density, \( m \) is mass, and \( V \) is volume.

\[ m = M \times n \]

where \( M \) is molar mass and \( n \) is the number of moles.

Now take the definition of density (Equation \ref{10.5.1})

\[ \rho = \frac{m}{V} \]

Keeping in mind \( m = M \times n \)...replace \((M \times n)\) for \((mass)\) within the density formula.

\[ \rho = \frac{M \times n}{V} \]
\[ \frac{\rho}{M} = \frac{n}{V} \]

Now manipulate the Ideal Gas Equation

\[ PV = nRT \]
\[ \frac{n}{V} = \frac{P}{RT} \]

\((n/V)\) is in both equations.

\[ \begin{align*}
\frac{n}{V} &= \frac{\rho}{M} \\
&= \frac{P}{RT}
\end{align*}\]

Now combine them please.

\[ \frac{\rho}{M} = \frac{P}{RT} \]

Isolate density.

\[ \rho = \frac{PM}{RT} \]

Step 4: Now plug in the information you have.
The density of a gas **increases** with increasing pressure and **decreases** with increasing temperature.

An example of varying density for a useful purpose is the hot air balloon, which consists of a bag (called the envelope) that is capable of containing heated air. As the air in the envelope is heated, it becomes less dense than the surrounding cooler air (Equation \ref{10.5.2}), which has enough lifting power (due to buoyancy) to cause the balloon to float and rise into the air. Constant heating of the air is required to keep the balloon aloft. As the air in the balloon cools, it contracts, allowing outside cool air to enter, and the density increases. When this is carefully controlled by the pilot, the balloon can land as gently as it rose.

**Figure \((\text{PageIndex}{1})\):** A hot air balloon is inflated partially with cold air from a gas-powered fan, before the propane burners are used for final inflation.

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**Determining Gas Volumes in Chemical Reactions**

The ideal gas law can be used to calculate volume of gases consumed or produced. The ideal-gas equation frequently is used to interconvert between volumes and molar amounts in chemical equations.

**Example \((\text{PageIndex}{2A})\)**

What volume of carbon dioxide gas is produced at STP by the decomposition of 0.150 g \(\ce{CaCO_3(s)}\) via the equation:

\[
\ce{CaCO3(s) -> CaO(s) + CO2(g)}
\]

**SOLUTION**

Begin by converting the mass of calcium carbonate to moles.
\[
\frac{0.150\,\text{g}}{100.1\,\text{g/mol}} = 0.00150\,\text{mol}
\]

The stoichiometry of the reaction dictates that the number of moles \(\text{CaCO}_3\) decomposed equals the number of moles \(\text{CO}_2\) produced. Use the ideal-gas equation to convert moles of \(\text{CO}_2\) to a volume.

\[
\begin{align*}
V &= \frac{nRT}{R} \\
&= \frac{(0.00150\,\text{mol})\left(0.08206\,\text{L} \cdot \text{atm}/\text{mol} \cdot \text{K}\right)(273.15\,\text{K})}{1\,\text{atm}} \\
&= 0.0336\,\text{L} \quad \text{or} \quad 33.6\,\text{mL}
\end{align*}
\]

Example \(\PageIndex{2B}\)

A 3.00 L container is filled with \(\text{Ne(g)}\) at 770 mmHg at 27\(^\circ\)C. A \(0.633\,\text{g}\) sample of \(\text{CO}_2\) vapor is then added. What is the partial pressure of \(\text{CO}_2\) and \(\text{Ne}\) in atm? What is the total pressure in the container in atm?

**SOLUTION**

*Step 1: Write down all given information, and convert as necessary.*

Before:

- \(P = 770\,\text{mmHg} \rightarrow 1.01\,\text{atm}\)
- \(V = 3.00\,\text{L}\)
- \(n_{\text{Ne}} = ?\)
- \(T = 27^\circ\,\text{C} \rightarrow (300\,\text{K})\)

Other Unknowns: \(n_{\text{CO}_2} = ?\)

\[
\begin{align*}
n_{\text{CO}_2} &= 0.633\,\text{g} \times \frac{1\,\text{mol}}{44\,\text{g}} = 0.0144\,\text{mol} \quad \text{CO}_2
\end{align*}
\]

*Step 2: After writing down all your given information, find the unknown moles of Ne.*

\[
\begin{align*}
n_{\text{Ne}} &= \frac{PV}{RT} \\
&= \frac{(1.01\,\text{atm})(3.00\,\text{L})}{(0.08206\,\text{atm} \cdot \text{L} / \text{mol} \cdot \text{K})(300\,\text{K})} \\
&= 0.123 \,\text{mol}
\end{align*}
\]

Because the pressure of the container before the \(\text{CO}_2\) was added contained only \(\text{Ne}\), that is your partial pressure of \(\text{Ne}\). After converting it to atm, you have already answered part of the question!

\[
P_{\text{Ne}} = 1.01\,\text{atm}
\]

*Step 3: Now that have pressure for Ne, you must find the partial pressure for \(\text{CO}_2\). Use the ideal gas equation.*

\[
\begin{align*}
\frac{P_{\text{Ne}}V}{n_{\text{Ne}}RT} &= \frac{P_{\text{CO}_2}V}{n_{\text{CO}_2}RT} \\
\text{but because both gases share the same Volume (}\langle \text{V} \rangle\text{) and Temperature (}\langle \text{T} \rangle\text{) and since the Gas Constant (}\langle \text{R} \rangle\text{) is constants, all three terms cancel and can be removed them from the equation.}
\end{align*}
\]

\[
\frac{\text{dfrac}{P}}{\text{dfrac}{n_{\text{Ne}}}} = \text{dfrac}{P}{n_{\text{CO}_2}}
\]

\[4\]
This is the partial pressure $\ce{CO_2}$.

Step 4: Now find total pressure.

\[
\begin{align*}
P_{\text{total}} &= P_{\text{Ne}} + P_{\text{CO}_2} \\
&= 1.01 \; \text{atm} + 0.118 \; \text{atm} \\
&\approx 1.13 \; \text{atm} \; \text{(with appropriate significant figures)}
\end{align*}
\]

Example (Page Index 2C): Sulfuric Acid

Sulfuric acid, the industrial chemical produced in greatest quantity (almost 45 million tons per year in the United States alone), is prepared by the combustion of sulfur in air to give SO$_2$, followed by the reaction of SO$_2$ with O$_2$ in the presence of a catalyst to give SO$_3$, which reacts with water to give H$_2$SO$_4$. The overall chemical equation is as follows:

\[
\ce{2S(s) + 3O2(g) + 2H2O(l) -> 2H2SO4(aq)}
\]

What volume of O$_2$ (in liters) at 22°C and 745 mmHg pressure is required to produce 1.00 ton (907.18 kg) of H$_2$SO$_4$?

**Given:** reaction, temperature, pressure, and mass of one product

**Asked for:** volume of gaseous reactant

**Strategy:**

A Calculate the number of moles of H$_2$SO$_4$ in 1.00 ton. From the stoichiometric coefficients in the balanced chemical equation, calculate the number of moles of O$_2$ required.

B Use the ideal gas law to determine the volume of O$_2$ required under the given conditions. Be sure that all quantities are expressed in the appropriate units.

**Solution:**

mass of H$_2$SO$_4$ → moles H$_2$SO$_4$ → moles O$_2$ → liters O$_2$

A We begin by calculating the number of moles of H$_2$SO$_4$ in 1.00 ton:

\[
\frac{907.18 \times 10^3 \; \text{g} \; \text{H}_2\text{SO}_4}{(2 \times 1.008 + 32.06 + 4 \times 16.00) \; \text{g/mol}} = 9250 \; \text{mol} \; \text{H}_2\text{SO}_4
\]

We next calculate the number of moles of O$_2$ required:

\[
9250 \; \text{mol} \; \text{H}_2\text{SO}_4 \times \frac{3 \; \text{mol} \; \text{O}_2}{2 \; \text{mol} \; \text{H}_2\text{SO}_4} = 1.389 \times 10^4 \; \text{mol} \; \text{O}_2
\]

B After converting all quantities to the appropriate units, we can use the ideal gas law to calculate the volume of O$_2$:

\[
V = \frac{nRT}{P} = \frac{1.389 \times 10^4 \; \text{mol} \times 0.08206 \; \text{L} \cdot \text{atm/mol} \cdot \text{K}}{745 \; \text{mmHg} \times \frac{1 \; \text{atm}}{760 \; \text{mmHg}}} = 3.43 \times 10^5 \; \text{L}
\]
The answer means that more than 300,000 L of oxygen gas are needed to produce 1 ton of sulfuric acid. These numbers may give you some appreciation for the magnitude of the engineering and plumbing problems faced in industrial chemistry.

Exercise \(\PageIndex{2}\)

Charles used a balloon containing approximately 31,150 L of \(\text{H}_2\) for his initial flight in 1783. The hydrogen gas was produced by the reaction of metallic iron with dilute hydrochloric acid according to the following balanced chemical equation:

\[
\text{Fe(s) + 2 HCl(aq) \rightarrow H}_2(\text{g}) + \text{FeCl}_2(\text{aq})
\]

How much iron (in kilograms) was needed to produce this volume of \(\text{H}_2\) if the temperature was 30°C and the atmospheric pressure was 745 mmHg?

Answer

68.6 kg of Fe (approximately 150 lb)

Example \(\PageIndex{3}\): Emergency Air bags

Sodium azide \((\text{NaN}_3)\) decomposes to form sodium metal and nitrogen gas according to the following balanced chemical equation:

\[
\text{2NaN}_3 \rightarrow 2\text{Na(s)} + 3\text{N}_2(\text{g})
\]

This reaction is used to inflate the air bags that cushion passengers during automobile collisions. The reaction is initiated in air bags by an electrical impulse and results in the rapid evolution of gas. If the \(\text{N}_2\) gas that results from the decomposition of a 5.00 g sample of \((\text{NaN}_3)\) could be collected by displacing water from an inverted flask, what volume of gas would be produced at 21°C and 762 mmHg?

**Given:** reaction, mass of compound, temperature, and pressure

**Asked for:** volume of nitrogen gas produced

**Strategy:**

A Calculate the number of moles of \(\text{N}_2\) gas produced. From the data in Table 10.5.4, determine the partial pressure of \(\text{N}_2\) gas in the flask.

B Use the ideal gas law to find the volume of \(\text{N}_2\) gas produced.

**Solution:**

A Because we know the mass of the reactant and the stoichiometry of the reaction, our first step is to calculate the number of moles of \(\text{N}_2\) gas produced:

\[
\frac{5.00\text{ g}\text{NaN}_3}{(22.99+3\times14.01)\text{ g/mol}}\times\frac{3\text{ mol}\text{N}_2}{2\text{ mol}\text{NaN}_3}=0.115\text{ mol; }\text{N}_2
\]
The pressure given (762 mmHg) is the total pressure in the flask, which is the sum of the pressures due to the N\(_2\) gas and the water vapor present. Table 10.5.4 tells us that the vapor pressure of water is 18.65 mmHg at 21°C (294 K), so the partial pressure of the N\(_2\) gas in the flask is only

\[
\text{[\text{m}(762 - 18.65)] \times \frac{1\; \text{atm}}{760\; \text{atm}} = 743.4\; \text{mmHg} \times \frac{1\; \text{atm}}{760\; \text{atm}} = 0.978\; \text{atm}}.
\]

B Solving the ideal gas law for \(V\) and substituting the other quantities (in the appropriate units), we get

\[
V = \frac{nRT}{P} = \frac{0.115\; \text{mol} \times 0.08206\; \frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}} \times 294\; \text{K}}{0.978\; \text{atm}} = 2.84\; \text{L}.
\]

Exercise

A 1.00 g sample of zinc metal is added to a solution of dilute hydrochloric acid. It dissolves to produce H\(_2\) gas according to the equation

\[
\text{Zn(s) + 2 HCl(aq) → H}_2\text{(g) + ZnCl}_2\text{(aq)}.}
\]

The resulting H\(_2\) gas is collected in a water-filled bottle at 30°C and an atmospheric pressure of 760 mmHg. What volume does it occupy?  
Answer

0.397 L

Summary

The relationship between the amounts of products and reactants in a chemical reaction can be expressed in units of moles or masses of pure substances, of volumes of solutions, or of volumes of gaseous substances. The ideal gas law can be used to calculate the volume of gaseous products or reactants as needed. In the laboratory, gases produced in a reaction are often collected by the displacement of water from filled vessels; the amount of gas can then be calculated from the volume of water displaced and the atmospheric pressure. A gas collected in such a way is not pure, however, but contains a significant amount of water vapor. The measured pressure must therefore be corrected for the vapor pressure of water, which depends strongly on the temperature.