Within the context of a harmonic bath, the term "random force" is something of a misnomer, since \( R(t) \) is completely deterministic and not random at all!!! We will return to this point momentarily, however, let us examine particular features of \( R(t) \) from its explicit expression from the harmonic bath dynamics. Note, first of all, that it does not depend on the dynamics of the system coordinate \( \underline{q} \) (except for the appearance of \( q(0) \)). In this sense, it is independent or "orthogonal" to \( \underline{q} \) within a phase space picture. From the explicit form of \( R(t) \), it is straightforward to see that the correlation function

\[
\langle \dot{q}(0)R(t) \rangle = 0
\]

i.e., the correlation function of the system velocity \( \underline{\dot{q}} \) with the random force is 0. This can be seen by substituting in the expression for \( R(t) \) and integrating over initial conditions with a canonical distribution weighting. For certain potentials \( \phi(q) \) that are even in \( \underline{q} \) (such as a harmonic oscillator), one can also show that

\[
\langle q(0)R(t) \rangle = 0
\]

Thus, \( R(t) \) is completely uncorrelated from both \( \underline{q} \) and \( \underline{\dot{q}} \), which is a property we might expect from a truly random process. In fact, \( R(t) \) is determined by the detailed dynamics of the bath. However, we are not particularly interested or able to follow these detailed dynamics for a large number of bath degrees of freedom. Thus, we could just as well model \( R(t) \) by a completely random process (satisfying certain desirable features that are characteristic of a more general bath), and, in fact, this is often done. One could, for example, postulate that \( R(t) \) act over a maximum time \( t_{\text{max}}/\Delta t \) at discrete points in time \( k \Delta t \), giving \( N=t_{\text{max}}/\Delta t \) values of \( R_k=R(k\Delta t) \), and assume that \( R_k \) takes the form of a *gaussian random process*:

\[
R_k = \sum_{j=1}^N \left[ a_je^{2\pi ijk/N} + b_je^{-2\pi ijk/N} \right]
\]

where the coefficients \( \{a_j\} \) and \( \{b_j\} \) are chosen at random from a gaussian distribution function. This might be expected to be suitable for a bath of high density, where strong collisions between the system and a bath particle are essentially nonexistent, but where the system only sees feels the relatively "soft" fluctuations of the less mobile bath. For a low density bath, one might try modeling \( R(t) \) as a Poisson process of very strong collisions.

Whatever model is chosen for \( R(t) \), if it is a truly random process that can only act at discrete points in time, then the GLE takes the form of a stochastic (based on random numbers) integro-differential equation. There is a whole body of mathematics devoted to the properties of such equations, where heavy use of an Itô calculus is made.

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**Contributors and Attributions**

Mark Tuckerman (New York University)