We construct a Cartesian space in which each of the \( (6N) \) coordinates and momenta is assigned to one of \( (6N) \) mutually orthogonal axes. Phase space is, therefore, a \( (6N) \) dimensional space. A point in this space is specified by giving a particular set of values for the \( (6N) \) coordinates and momenta. Denote such a point by

\[
(x = (p_1, \cdots , p_N, r_1, \cdots , r_N ))
\]

\( (x) \) is a \( (6N) \) dimensional vector. Thus, the time evolution or trajectory of a system as specified by Hamilton's equations of motion, can be expressed by giving the phase space vector, \( (x) \) as a function of time.

**Figure:** Evolution of an ensemble of classical systems in phase space (top). The systems are a massive particle in a one-dimensional potential well (red curve, lower figure). The initially compact ensemble becomes swirled up over time.

The law of conservation of energy, expressed as a condition on the phase space vector:

\[
(H(x(t)) = \text{const} = E)
\]

defines a \( (6N - 1) \) dimensional hypersurface in phase space on which the trajectory must remain.

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