Minerals can usually be identified by their **specific gravity** (SG). Because samples of the same mineral may appear quite different, as these samples of cinnabar show, a quantitative, objective measurement like SG may help confirm their identity. SG might also be used (as Archimedes did) to distinguish pure gold (SG = 19.3) from 14K gold (SG = 13.3) or pyrite (SG = 5.1). Some specific gravities of minerals are shown below. Online mineral vendors sell equipment for determining specific gravity, but it is simple to do in the lab. Specific gravity is related to density, but it is unitless. Earlier we showed how *unity factors* can be used to express quantities in different units of the same parameter. For example, a density can be expressed in g/cm$^3$ or lb/ft$^3$. Now we will also how *conversion factors* representing mathematical functions, like $D = m/v$, can be used to transform quantities into different parameters. Unity factors and conversion factors are conceptually different, and we'll see that the “dimensional analysis” we develop for unit conversion problems must be used with care in the case of functions.

![Cinnabar, HgS samples](image)

Specific Gravity (SG) is determined by attaching the sample to a string hung from a specially designed balance. The mass of the sample is determined, then a beaker of water is raised from below the sample until the sample is submerged, and the sample is reweighed. The sample weighs less in water by an amount equal to the mass of the water displaced by the sample. This is *Archimedes Principle*. The SG is calculated with the equation:

$$\text{SG} = \frac{\text{mass in air}}{\text{mass in air} - \text{mass in water}}$$

*The mass of the sample does not change, so it would be more appropriate to say weight. Here "mass" should be understood as "mass equivalent" of the weight, since balances are calibrated in grams, a unit of mass. Weights are technically measured in Newtons.*

The denominator of the equation represents the bouyancy, which is the mass equivalent of water displaced. The equation shows that SG is dimensionless, because both the numerator and denominator of the right side have units of 1.
mass. How can SG be similar to density?

Suppose we weigh a mineral sample and find that it weighs 78.246 g in air and 63.089 g in water. Let’s see how the SG is determined.

The denominator (mass in air - mass in water) = 78.246-63.089 = 15.157 g. This is the mass of water displaced. The volume of the sample is the same as the volume of water displaced, and we can calculate the volume by using the density of water as a conversion factor, as follows:

Remember the mathematical function which defines density:
\[
\text{Density} = \dfrac{\text{mass}}{\text{volume}} \text{ or } \rho = \dfrac{m}{V}
\]

The formula which defines density can also be used to convert the mass of a sample to the corresponding volume. If both sides of Eq. (1.2) are multiplied by \(1/\rho\), we have
\[
\dfrac{1}{\rho} \times m = V \times \rho \times \dfrac{1}{\rho} = V
\]
\[
V = m \times \dfrac{1}{\rho}
\]

If the measurement is made at 25 °C, the volume is calculated from the density of water (found in tables like the one below):
\[
V = \dfrac{m}{\rho} = \dfrac{15.157 \text{ g}}{0.9922 \text{ g/cm}^3} = 15.202 \text{ cm}^3
\]

Note that the SG is relative to water at a particular temperature, which must be specified.

Table (Page Index{1}) Density of Water

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Density/g/cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.9999</td>
</tr>
<tr>
<td>4</td>
<td>1.0000</td>
</tr>
<tr>
<td>20</td>
<td>0.9982</td>
</tr>
<tr>
<td>22</td>
<td>0.997735</td>
</tr>
<tr>
<td>25</td>
<td>0.9970479</td>
</tr>
<tr>
<td>40</td>
<td>0.9922</td>
</tr>
</tbody>
</table>

Now that the volume of the sample is known, it can be used to calculate the density of the mineral:
\[
\rho = \dfrac{m}{V} = \dfrac{78.246 \text{ g}}{15.202 \text{ cm}^3} = 5.147 \text{ g/cm}^3
\]

It appears that the sample is Pyrite. The Examples below give the data that would be expected for pure gold and 14K gold.

Note that if the density of water is used explicitly in the equation
\[
\rho = \dfrac{\text{mass in air}}{\dfrac{\text{mass in air - mass in water}}{\dfrac{m}{V}}}\]

then the SG becomes an absolute density with units g/cm³. But a SG can be obtained, even if the density of water is unknown, from just mass measurements. In that case, it’s a relative, unitless value \(\dfrac{\text{density of mineral}}{\text{density of water}}\).
Table \(\PageIndex{2}\) SG of Minerals

<table>
<thead>
<tr>
<th>Mineral</th>
<th>SG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borax</td>
<td>1.7</td>
</tr>
<tr>
<td>Ulexite</td>
<td>1.9</td>
</tr>
<tr>
<td>Kernite</td>
<td>1.95</td>
</tr>
<tr>
<td>Gypsum</td>
<td>2.3-2.4</td>
</tr>
<tr>
<td>Bauxite</td>
<td>2.4-2.6</td>
</tr>
<tr>
<td>Quartz</td>
<td>2.65</td>
</tr>
<tr>
<td>Calcite</td>
<td>2.7</td>
</tr>
<tr>
<td>Fluorite</td>
<td>3.0-3.2</td>
</tr>
<tr>
<td>Stibnite</td>
<td>4.6</td>
</tr>
<tr>
<td>Zircon</td>
<td>4.6-4.7</td>
</tr>
<tr>
<td>Pyrite</td>
<td>4.9-5.2</td>
</tr>
<tr>
<td>Hematite</td>
<td>4.9-5.3</td>
</tr>
<tr>
<td>Galena</td>
<td>7.4-7.6</td>
</tr>
<tr>
<td>Cinnabar</td>
<td>8.0-8.2</td>
</tr>
<tr>
<td>Native Copper</td>
<td>8.9</td>
</tr>
<tr>
<td>Native Gold</td>
<td>19.32</td>
</tr>
</tbody>
</table>

Example \(\PageIndex{1}\): Weight of Gold

If a sample of pure gold weighs the same as the pyrite sample above, 78.246 g in air, how much will it weigh submerged in a SG balance at 25 °C?

**Solution**

To solve this problem, we need the volume of gold, which we can get as above from the density:

\[
\text{V} = \dfrac{m}{\rho} = \dfrac{78.246 \text{ g}}{19.32 \text{ g/cm}^3} = 4.050 \text{ cm}^3
\]

This is the volume of water displaced. Now we need the mass of water displaced, which is the bouyancy, or the difference in mineral mass in air and water.

If we multiply both sides of Eq. 1.1 by \(V\), we obtain

\[
\text{m} = \text{V} \times \rho
\]

or

\[\text{mass} = \text{volume} \times \text{density}\]

So for a volume of 4.050 cm\(^3\) and \(\rho = 0.9971\) g/cm\(^3\), we calculate a mass of

\[
\text{m} = 4.050 \text{ cm}^3 \times 0.9971 \dfrac{\text{g}}{\text{cm}^3} = 4.038 \text{ g}
\]

so its submerged mass will be 78.24 g - mass in water = 4.035 g; mass in water = 74.208 g.

Notice that we used the mathematical function \(D = m/V\) to convert parameters from mass to volume or vice versa in these examples. How does this differ from the use of unity factors to change units of one parameter?

**An Important Caveat:** First, notice that we cannot always depend on cancelling units to solve a problem. The SG has no units, so we need to know the mathematical function to solve problems. But in all cases, it is important to know the mathematical function, as well as the unit factor method, as the following example shows.

A mistake sometimes made by beginning students is to confuse density with concentration, which also may have units of g/cm\(^3\). By dimensional analysis, this looks perfectly fine. To see the error, we must understand the meaning of the function

\[
C = \dfrac{m}{V}
\]

In this case, \(V\) refers to the volume of a solution, which contains both a solute and solvent.
Given a concentration of an alloy is 10g gold in 100 cm$^3$ of alloy, we see that it is wrong (although dimensionally correct as far as conversion factors go) to **incorrectly** calculate the volume of gold in 20 g of the alloy as follows:

\[ 20 \text{ g} \times \dfrac{100 \text{ cm}^3}{10 \text{ g}} = 200 \text{ cm}^3 \]

It is only possible to calculate the volume of gold if the density of the alloy is known, so that the volume of alloy represented by the 20 g could be calculated. This volume multiplied by the concentration gives the mass of gold, which then can be converted to a volume with the density function.

The bottom line is that using a simple unit cancellation method does not always lead to the expected results, unless the mathematical function on which the conversion factor is based is fully understood.

**Example \(\PageIndex{2}\): Volume of Gold**

Gold can be extracted from low grade gold ore by the "cyanide process". A gold ore with a concentration of 0.060 g / cm$^3$ has a density of 8.25 g / cm$^3$. What is the volume of gold (D = 19.32 g / cm$^3$) in 100 cm$^3$ of the ore?

**Answer**

The volume of 100 g of ore is

\[ V = \dfrac{m}{D} = \dfrac{100 \text{ g}}{8.25 \text{ g/cm}^3} = \text{12.12 cm}^3 \]

The mass of gold in this volume is

\[ m = V \times C = \text{12.12 cm}^3 \times \text{0.060 g/cm}^3 = \text{0.727 g} \]

The volume of gold =

\[ \dfrac{m}{D} = \dfrac{0.727 \text{ g}}{19.32 \text{ g/cm}^3} = \text{0.0376 cm}^3. \]

Note that we cannot calculate the volume of gold by

\[ \dfrac{8.25 \text{ g/cm}^3 \times 100 \text{ cm}^3}{19.32 \text{ g/cm}^3} = \text{42.70 cm}^3 \]

even though this is dimensionally correct.

Note that this result required when to use the function \( C = \dfrac{m}{V} \), and when to use the function \( D = \dfrac{m}{V} \) as conversion factors. Pure dimensional analysis could not reliably give the answer, since both functions have the same dimensions.

The two calculations just done show that density is a conversion factor which changes volume to mass, and the reciprocal of density is a conversion factor changing mass into volume. This can be done because of the mathematical formula, \( D = \dfrac{m}{V} \), which relates density, mass, and volume. Algebraic manipulation of this formula gave us expressions for mass and for volume, and we used them to solve our problems. If we understand the function \( D = \dfrac{m}{V} \) and heed the caveat above, we can devise appropriate conversion factors by unit cancellation, as the following example shows.

**Example \(\PageIndex{1}\): Volume of Mercury**
A student weighs 98.0 g of mercury. If the density of mercury is 13.6 g/cm$^3$, what volume does the sample occupy?

**Solution**

We know that volume is related to mass through density. 

Therefore

\[ V = \frac{m}{\rho} \]

Since the mass is in grams, we need to get rid of these units and replace them with volume units. This can be done if the reciprocal of the density is used as a conversion factor. This puts grams in the denominator so that these units cancel:

\[ V = \frac{98.0 \text{ g}}{13.6 \text{ g/cm}^3} = 7.21 \text{ cm}^3 \]

If we had multiplied by the density instead of its reciprocal, the units of the result would immediately show our error:

\[ V = 98.0 \text{ g} \times \frac{13.6 \text{ g}}{1 \text{ cm}^3} = \text{1.333 g}^2/\text{cm}^3 \]

Note that there was no cancellation!

It is clear that square grams per cubic centimeter are not the units we want.

Using a conversion factor is very similar to using a unity factor—we know the factor is correct when units cancel appropriately. A conversion factor is not unity, however. Rather it is a physical quantity (or the reciprocal of a physical quantity) which is related to the two other quantities we are interconverting. The conversion factor works because of that relationship [Eqs. (1.1), (1.2), and (1.3) in the case of density, mass, and volume], not because it is equal to one. Once we have established that a relationship exists, it is no longer necessary to memorize a mathematical formula. The units tell us whether to use the conversion factor or its reciprocal. Without such a relationship, however, mere cancellation of units does not guarantee that we are doing the right thing.

A simple way to remember relationships among quantities and conversion factors is a “road map” of the type shown below:

\[ \text{First quantity } \overset{\text{conversion factor}}{\longleftrightarrow} \text{ second quantity} \]

This indicates that the mass of a particular sample of matter is related to its volume (and the volume to its mass) through the conversion factor, density. The double arrow indicates that a conversion may be made in either direction, provided the units of the conversion factor cancel those of the quantity which was known initially. In general the road map can be written

\[ \text{First quantity } \overset{\text{conversion factor}}{\longleftrightarrow} \text{ second quantity} \]

As we come to more complicated problems, where several steps are required to obtain a final result, such road maps will become more useful in charting a path to the solution.
Example \((\PageIndex{4})\): Black Ironwood

Black ironwood has a density of \(67.24 \text{ lb/ft}^3\). If you had a sample whose volume was 47.3 ml, how many grams would it weigh? (1 lb = 454 g; 1 ft = 30.5 cm).

**Solution**

The road map

\[
\text{m} \overset{\rho}{\longleftrightarrow} V \nonumber
\]

tells us that the mass of the sample may be obtained from its volume using the conversion factor, density. Since milliliters and cubic centimeters are the same, we use the SI units for our calculation:

\[
\text{Mass} = \text{47.3 cm}^3 \times \dfrac{\text{67.24 lb}}{\text{1 ft}^3} \nonumber
\]

Since the volume units are different, we need a unity factor to get them to cancel:

\[
\text{m} = \text{47.3 cm}^3 \times \dfrac{\text{1 ft}^3}{\text{30.5 cm}^3} \times \dfrac{\text{67.24 lb}}{\text{1 ft}^3} \times \dfrac{\text{454 g}}{\text{1 lb}} = \text{50.9 g} \nonumber
\]

In subsequent chapters we will establish a number of relationships among physical quantities. Formulas will be given which define these relationships, but we do not advocate slavish memorization and manipulation of those formulas. Instead we recommend that you remember that a relationship exists, perhaps in terms of a road map, and then adjust the quantities involved so that the units cancel appropriately. Such an approach has the advantage that you can solve a wide variety of problems by using the same technique.

**References**


**Contributors and Attributions**

- Ed Vitz (Kutztown University), John W. Moore (UW-Madison), Justin Shorb (Hope College), Xavier Prat-Resina (University of Minnesota Rochester), Tim Wendorff, and Adam Hahn.