Mechanical Energies

Sir Issac Newton (1642-1727) was the culminating figure of the scientific revolution during the 17th century; few matched him.

- Newton's 1st law defines mass \( m \) as a measure of inertia. The SI unit for mass is in kg.
- His 2nd law gives the acceleration \( a \) imparted to a body by a force \( F \)

\[
F = ma
\]

where \( F \) and \( a \) are vector quantities having directions and magnitudes. The SI units for \( a \) are m s\(^{-2}\), and thus this law defines force. The gravitational pull for 102 gram is 1 Newton (N = m kg s\(^{-2}\)).

- \textit{Work} \( (W) \) is defined as the force applied over a distance \( s \) along the direction of the force.

\[
W = F \cdot s
\]

- The gravitational force of a mass \( m \) on the Earth surface is \( g \cdot m \), \((g = 9.8 \text{ m s}^{-2})\). Thus the potential energy \( E_p \) of a mass located at a height \( h \) above a reference of zero is

\[
E_p = g \cdot m \cdot h \text{ m}^2 \text{ kg s}^{-2}
\]

- When the mass lost an altitude \( h \) and return to the zero level, all the potential energy is converted to kinetic energy \( E_k \).

\[
E_k = g \cdot m \cdot h = 1/2 m v^2
\]

where \( v \) is the velocity.

Example 1

A football player weighing 100 kg walks up to a height of 10 m. How much potential energy has he gained?

\textbf{SOLUTION}

\[
E_p = g \cdot m \cdot h \\
= 9.8 \cdot 100 \cdot 10 \text{ m s}^{-2} \text{ kg m} \\
= 9800 \text{ m}^2 \text{ kg s}^{-2} \text{ (same as J)}
\]

Example 2

A football player weighing 100 kg falls from a height of 10 m. What is his velocity when he falls to ground zero?

\textbf{SOLUTION}

From the results of the previous example, we have

\[
9800 \text{ m}^2 \text{ kg s}^{-2} = 1/2 m v^2
\]
Thus,

\[ v = (2 \times 9800 / 100)^{1/2} = 14 \text{ m s}^{-1} \]

**DISCUSSION**

Does the velocity depends on mass of the person?

No! since \( m g h = \frac{1}{2} m v^2 \)

\[ v = (2 g h)^{1/2} \]

Example 3

How much water is required for the production of 1000 kJ electric energy from a water fall whose height is 100 m? Assume all the potential energy is converted to electric energy in the process. How much water is required if the efficiency of electric energy generation is 70%?

**SOLUTION**

Assume the mass of the water to be \( m \) kg, then we have

\[ m g h = 1000 \text{ J} \]

\[ m = \frac{1000000 \text{ J}}{(9.8 \text{ m s}^{-2} \times 100 \text{ m})} \]

\[ m = 1020 \text{ kg} \]

If the efficiency is 70% the amount of water required is

\[ \text{amount of water} = \frac{1020 \text{ kg}}{0.70} = 146 \text{ kg}. \]

**Contributors**

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