Skills to Develop

- Explain the spectrum from hydrogen gas.
- Describe Rydberg's theory for the hydrogen spectra.
- Interpret the hydrogen spectrum in terms of the energy states of electrons.

Hydrogen Spectra

Lasers emit radiation which is composed of a single wavelength. However, most common sources of emitted radiation (i.e. the sun, a lightbulb) produce radiation containing many different wavelengths. When the different wavelengths of radiation are separated from such a source a spectrum is produced. A rainbow represents the spectrum of wavelengths of light contained in the light emitted by the sun. Sun light passing through a prism (or raindrops) is separated into its component wavelengths and is made up of a continuous spectrum of wavelengths (from red to violet); there are no gaps.

Not all radiation sources emit a continuous spectrum of wavelengths of light. When high voltage is applied to a glass tube containing various gasses under low pressure different colored light is emitted: neon gas produces a red-orange glow and sodium gas produces a yellow glow. When such light is passed through a prism only a few wavelengths are present in the resulting spectra that appear as lines separated by dark areas, and thus are called line spectra.

Hydrogen, the simplest but the most abundant element in the universe, is also the most studied element. During the early development of science, people had been investigating light emitted by heated tubes of hydrogen gas. When these lights passed prisms, they saw some lines in the visible region. When the spectrum emitted by hydrogen gas was passed through a prism and separated into its constituent wavelengths four lines appeared at characteristic wavelengths in the visible spectral range: 656 nm, 486 nm, 434 nm, and 410 nm.
These lines are shown here together with lines emitted by hot gases of \(\ce{Hg}\) and \(\ce{He}\). These lines are called the **Balmer Series**, because Balmer saw some regularity in their wavelength, and he has given a formula to show the regularity.

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### The Balmer Series

J.J. Balmer analyzed these lines and identified the following relationship:

\[
\lambda = 364.56 \frac{n_2^2}{n^2 - 2^2} \text{ nm}
\]

for the regularity in terms of integers \(n_2\). For some integers of \(n_\geq 3\), you can confirm the \(\lambda\) to be

<table>
<thead>
<tr>
<th>(n_2)</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda)</td>
<td>656</td>
<td>486</td>
<td>434</td>
<td>410</td>
<td>397</td>
<td>389</td>
<td>383</td>
<td>380</td>
</tr>
<tr>
<td>color</td>
<td>red</td>
<td>teal</td>
<td>blue</td>
<td>indigo</td>
<td>viol</td>
<td>not visible</td>
<td>not visible</td>
<td>not visible</td>
</tr>
</tbody>
</table>

As the \(n_2\) increase, the lines are getting closer together. If you plot the lines according to their \(\lambda\) on a linear scale, you will get the appearance of a spectrum as observed by experimentalists; these lines are called the **Balmer series**.

### The Rydberg Formula

Rydberg inverted both sides of Balmer's formula and gave

\[
\frac{1}{\lambda} = R_{\text{H}} \left( \frac{1}{2^2} - \frac{1}{n^2} \right)
\]

This is known as the **Rydberg formula**, and \(R\) is known as the **Rydberg constant**; its numerical values depends on the units used

\[
\begin{align*}
R_{\text{H}} &= 0.010972 \text{ nm}^{-1} \\
&= 10972 \text{ mm}^{-1} \\
&= 109721 \text{ cm}^{-1} \\
&= 10972130 \text{ m}^{-1}
\end{align*}
\]

This formula shows that if you plot \(\tilde{\nu} = \frac{1}{\lambda}\) vs. \(1/n^2\), you will get a straight line.
Other Series

The results given by Balmer and Rydberg for the spectrum in the visible region of the electromagnetic radiation start with \( n = 3 \), and the other integer is 2. Is there a series with the following formula?

\[
\frac{1}{\lambda} = R_{\text{H}} \left( \frac{1}{1^2} - \frac{1}{n^2} \right)
\]

If the law of nature is simple and regular, a series should exist, and the values for \( n \) and wavenumber (\( \nu_n \)) should be:

<table>
<thead>
<tr>
<th>( n_2 )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda ) (nm)</td>
<td>121</td>
<td>102</td>
<td>97</td>
<td>94</td>
<td>...</td>
</tr>
<tr>
<td>( \widetilde{\nu} ) (cm(^{-1}))</td>
<td>82,2291</td>
<td>97,530</td>
<td>102,864</td>
<td>105,332</td>
<td>...</td>
</tr>
</tbody>
</table>

Do you know in what region of the electromagnetic radiation these lines are? Of course, these lines are in the UV region, and they are not visible, but they are detected by instruments; these lines form a Lyman series. The existences of the Lyman series and Balmer’s series suggest the existence of more series, and a generalized formula is suggested.

\[
\nu_n = R_{\text{H}} \left( \frac{1}{n_{\text{f}}^2} - \frac{1}{n_{\text{i}}^2} \right)
\]

Actually, the series with \( n_2^2 = 3 \), and \( n_1^2 = 4, 5, 6, 7, \ldots \) is called Pashen series.

Questions

1. **Calculate the wave numbers of the lines with the longest wavelength in the Pashen series.**

   Hint: 5334 /cm

   **Discussion:**
   Calculate the wavelength for three lines in this series. What region are these lines?

2. **Draw an energy level diagram to show the transition for the emission of the various series of lines by hydrogen.**

   Hint: See the discussion below:

   **Discussion:**
   A diagram showing the energy levels is shown here.

   ```
   ====
   ..... very closely spaced lines
   ---- n = 3
   ---- n = 2
   ```
Interpretation of the hydrogen spectrum led to the development of quantum mechanics.

3. **Among all possible photons emitted by hydrogen atoms, what is the shortest wavelength possible?**

   Hint: The shortest wavelength = 1/R, where R is the Rydberg constant.

   **Discussion:**
   The photo with the highest energy from a hot hydrogen gas is in the Lyman series. The wavelength is about 91 nm. Confirm this by using the following equations.

   \[
   \omega_n = R \left( \frac{1}{n_{f}^2} - \frac{1}{n_{i}^2} \right)
   \]

   Spectra of Elements shows the spectra of elements in the periodic table.

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**Contributors**

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