It is finally time to turn our attention to pH. pH is just another way to express \([H^+]\), the hydrogen ion concentration of an acidic or basic solution.

Hydrogen acid concentrations are often small numbers, such as \(1.3 \times 10^{-3}\). pH is a method of transforming this number into something that is a little easier to work with.

In math class you may have learned about logarithms - log for short. We'll leave the definitions of logs to math and just work with how to find them here.

Get your calculators out. Different calculators work in slightly different ways, and it will be VERY IMPORTANT for you to know how to use yours when working with logs.

**What about pH?**

\[
\text{pH} = -\log[H^+]
\]

Because hydrogen ion concentrations are generally less than one
(for example $1.3 \times 10^{-3}$), the log of the number will be a negative number.

To make pH even easier to work with, pH is defined as the negative log of $[H^+]$, which will give a positive value for pH.
Try the examples shown on the right.

Find the pH, given \([H^+]\).

Answers are shown, but be sure you are able to arrive at that answer with your calculator!

Notice the last example.
1.0 × 10^{-7} is the [H^+] in pure water. Pure water therefore has a pH of 7.

By looking at the [H^+] values in the table above, can you determine which solutions would be acidic, and which would
be bases?

• Number 1 and 4 are acids. In those, \([H^+]\) is greater than 1.0×10^{-7}.

• Numbers 2 and 3 are bases. In those solutions, \([H^+]\) is less than 1.0×10^{-7}.

Working with negative powers of 10 is not easy for many of us, so
some of you may be confused by trying to identify acids and bases based on $[H^+]$.

But if we use pH values instead we find it much easier to identify acids and bases.

<table>
<thead>
<tr>
<th>Acids</th>
<th>Bases</th>
<th>Neutral solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>pH &lt; 7</td>
<td>pH &gt; 7</td>
<td>pH = 7</td>
</tr>
</tbody>
</table>
The lower the pH, the stronger the acid
The higher the pH, the stronger the base

Examples of Calculating pH

1. Calculate the pH of a 0.01M HNO₃ solution?

   Solution:

   Begin finding pH by first finding [H⁺].

   You should recognize HNO₃ as a strong acid, meaning it breaks down completely into ions. Based on the balanced equation shown below we see that there is a 1:1 ratio between HNO₃ and H⁺, so [HNO₃] = [H⁺]:

   \[
   \text{HNO}_3(aq) \rightarrow \text{H}^+(aq) + \text{NO}_3^-(aq)
   \]

   \[
   \text{[H}^+] = 0.01
   \]

   \[
   \text{pH} = -\log([\text{H}^+])
   \]

   \[
   \text{pH} = -\log(0.01)
   \]

   \[
   \text{pH} = -(-2.0)
   \]

   \[
   \text{pH} = 2.0
   \]

2. Find the pH of a 0.01 M solution of ammonia.

   Ammonia is a weak base with \[\text{K}_b = 1.8 \times 10^{-5}\]

   Solution:

   This is a more difficult question because we have a weak base (it would be similar for a weak acid). Because weak acids and bases do not ionize completely, we must use \(K_b\) to determine ion concentrations. That will require a balanced equation.
When writing equations for weak bases (that don’t contain an OH\(^-\)), be sure to include water, H\(_2\)O as a reactant.

Remember that the base will \textit{gain} a hydrogen ion:

\[
\ce{NH_3(g) + H_2O(l) \rightarrow NH_4^+(aq) + OH^-(aq)}
\]

Since ammonia is a base, first calculate \([\text{OH}^-]\). Then use \(K_w\) to determine \([\text{H}^+]\)

\[
\text{Set up the } K_b\text{ equation: } \quad \ce{K_b} = \dfrac{[NH_4^+][OH^-]}{[NH_3]}
\]

Substitute values into the equation.
Let \(x\) equal the unknowns

\[
\begin{align*}
\text{Rearrange the equation} & \quad \ce{x^2} = (1.8 \times 10^{-5})(.01) \\
\text{Take the square root} & \quad x = 4.2 \times 10^{-4}
\end{align*}
\]

Next we calculate \([\text{H}^+]\):

\[
\begin{align*}
\text{Rearrange the equation} & \quad [\text{H}^+] = \dfrac{K_w}{[\text{OH}^-]} \\
\text{Substitute in known values and calculate } [\text{H}^+] & \quad [\text{H}^+] = 2.4 \times 10^{-11}
\end{align*}
\]

Finally, convert \([\text{H}^+]\) into pH:

\[
\begin{align*}
\text{Calculate pH} & \quad \text{pH} = -\log([\text{H}^+]) \\
\text{Calculate pH} & \quad \text{pH} = -\log(2.4 \times 10^{-11})
\end{align*}
\]
There is a way to simplify the last parts of this operation. In addition to pH, we can also define pOH:

\[
\text{pOH} = -\log[\text{OH}^-]
\]

For bases, once we find [OH\(^-\)] for a base, we can quickly determine pOH:

Next we make use of the following easy-to-remember relationship:

\[
\text{pH} + \text{pOH} = 14
\]

Does the number 14 ring a bell?

Remember \[
\text{K}_w = 1.0 \times 10^{-14}
\]
The negative log of \[
1.0 \times 10^{-14} = 14
\]

Once we find pOH, it is a simple matter to find pH:
\[
\text{pH} + \text{pOH} = 14
\]

OR

\[
\text{pH} = 14 - \text{pOH}
\]

\[
\text{pH} = 14 - 3.4
\]

\[
\text{pH} = 10.6
\]

It doesn't matter which method you use to find \([H^+]\) and pH for a base - both will give you the same answer. Choose whichever method works best for you.

---

**Finding \([H^+]\) when you know pH**

Sometimes you need to work "backwards" - you know the pH of a solution and need to find \([H^+]\), or even the concentration of the acid solution. How do you do that?

To convert pH into \([H^+]\) involves taking the *antilog* of the negative value of pH.

\[
\text{[H}^+] = \text{antilog} (-\text{pH})
\]

As mentioned above, different calculators work slightly differently - make sure you can do the following calculations using *your* calculator. Practice as we go along . . .
**Example.** We have a solution with a pH = 8.3. What is \([H^+]\)?

With some calculators you will do things in the following order:

1. Enter 8.3 as a negative number (use the key with both the +/- signs, not the subtraction key)
2. Use your calculator's 2nd or Shift or INV (inverse) key to type in the symbol found above the LOG key. The shifted function should be \(10^x\).
3. You should get the answer \(5.0 \times 10^{-9}\)

Other calculators require you to enter keys in the order they appear in the equation.

1. Use the Shift or second function to key in the \(10^x\) function.
2. Use the +/- key to type in a negative number, then type in 8.3
3. You should get the answer \(5.0 \times 10^{-9}\)

If neither of these methods work, try rearranging the order in which you type in the keys. Don't give up - you must master your calculator!

Here are two final examples. Think *carefully* and determine what, exactly, you are asked to find and the steps needed to get there.

1. Find the hydronium ion concentration in a solution with a pH of 12.6. Is this solution an acid or a base? How do you know?

   **Solution:**

   We can easily answer the second part of the question - the solution is a base because pH > 7.

   Next we need to find hydronium ion concentration, \([H_3O^+]\), which you should remember will be the same as \([H^+]\)

   To convert pH into \([H_3O^+]\):

   \[
   \begin{align*}
   \ce{[H_3O^+]} &= \text{antilog (-pH)} \\
   \text{antilog (-12.6)} &= 2.5 \times 10^{-13}
   \end{align*}
   \]

2. A 0.24M solution of the weak acid, \(H_2CO_3\), has a pH of 3.49. Determine \(K_a\) for \(H_2CO_3\) (carbonic acid).

   **Solution:**

   This is much more difficult. In order to determine \(K_a\) we need to know the concentrations of several things:
Begin with a balanced equation for the acid:

\[
\ce{H_2CO_3(aq) \rightleftharpoons H^+(aq) + HCO_3^-(aq)}
\]

Next set up the equilibrium expression, which will be needed to find \(K_a\):

\[
K_a = \frac{[H^+][HCO_3^-]}{[H_2CO_3]}
\]

Next find \([H^+]\) from pH.

\[
[H^+] = \text{antilog} (-pH)
\]

\[
= \text{antilog} (-3.49)
\]

\[
= 3.2 \times 10^{-4}
\]

The balanced equation tells us that \([H^+] = [HCO_3^-]\). Substitute values into our \(K_a\) expression and solve:

\[
K_a = \frac{[H^+][HCO_3^-]}{[H_2CO_3]}
\]

The question gave us the the concentration of the acid, \(H_2CO_3\), as 0.24 M

\[
= \frac{[3.2 \times 10^{-4}][3.2 \times 10^{-4}]}{[0.24]}
\]

\[
K_a = 4.3 \times 10^{-7} \text{ answer}
\]