None of the approaches we have described so far can adequately explain why some compounds are colored and others are not, why some substances with unpaired electrons are stable, and why others are effective semiconductors. These approaches also cannot describe the nature of resonance. Such limitations led to the development of a new approach to bonding in which electrons are not viewed as being localized between the nuclei of bonded atoms but are instead delocalized throughout the entire molecule. Just as with the valence bond theory, the approach we are about to discuss is based on a quantum mechanical model.

Previously, we described the electrons in isolated atoms as having certain spatial distributions, called orbitals, each with a particular orbital energy. Just as the positions and energies of electrons in atoms can be described in terms of atomic orbitals (AOs), the positions and energies of electrons in molecules can be described in terms of molecular orbitals (MOs). A particular spatial distribution of electrons in a molecule that is associated with a particular orbital energy—a spatial distribution of electrons in a molecule that is associated with a particular orbital energy. As the name suggests, molecular orbitals are not localized on a single atom but extend over the entire molecule. Consequently, the molecular orbital approach, called molecular orbital theory is a delocalized approach to bonding.

Although the molecular orbital theory is computationally demanding, the principles on which it is based are similar to those we used to determine electron configurations for atoms. The key difference is that in molecular orbitals, the electrons are allowed to interact with more than one atomic nucleus at a time. Just as with atomic orbitals, we create an energy-level diagram by listing the molecular orbitals in order of increasing energy. We then fill the orbitals with the required number of valence electrons according to the Pauli principle. This means that each molecular orbital can accommodate a maximum of two electrons with opposite spins.

### The Hydrogen Molecule-Ion

Molecular orbital theory is a conceptual extension of the orbital model, which was so successfully applied to atomic structure. As was once playfully remarked, "a molecule is nothing more than an atom with more nuclei." This may be overly simplistic, but we do attempt, as far as possible, to exploit analogies with atomic structure. Our understanding of atomic orbitals began with the exact solutions of a prototype problem—the hydrogen atom. We will begin our study of homonuclear diatomic molecules beginning with another exactly solvable prototype, the hydrogen molecule-ion \(H_{2}^{+}\). This species actually has a transient existence in electrical discharges through hydrogen gas and has been detected by mass spectrometry. It also has been detected in outer space. The Schrödinger equation for \(H_{2}^{+}\) can be solved exactly within the Born-Oppenheimer approximation. For fixed internuclear distance \(R\), this reduces to a problem of one electron in the field of two protons, designated A and B. We can write

\[
\left\{-\frac{1}{2}\nabla^2 - \frac{1}{r_A} - \frac{1}{r_B} + \frac{1}{R} \right\} \psi(r) = E \psi(r) \tag{1}
\]

where \(r_A\) and \(r_B\) are the distances from the electron to protons A and B, respectively. This equation was solved by Burrau (1927), after separating the variables in prolate spheroidal coordinates.

### Molecular Orbitals Involving Only \(ns\) Atomic Orbitals

We begin our discussion of molecular orbitals with the simplest molecule, \(H_2\), formed from two isolated hydrogen atoms, each with a \((1s^1)\) electron configuration. As discussed before, electrons can behave like waves. In the molecular orbital approach, the overlapping atomic orbitals are described by mathematical equations called wave functions. The 1s atomic
orbitals on the two hydrogen atoms interact to form two new molecular orbitals, one produced by taking the sum of the two H 1s wave functions, and the other produced by taking their difference:

\[
\begin{matrix}
\text{MO}(1) &= \text{AO(atom A)} & + & \text{AO(atom B)} \\
\text{MO}(1) &= \text{AO(atom A)} & - & \text{AO(atom B)}
\end{matrix}
\]

The molecular orbitals created from the sum and the difference of two wavefunctions (atomic orbitals) from Equation \(\ref{Eq1}\) are called Linear Combinations of Atomic Orbitals (LCAOs). A molecule must have as many molecular orbitals as there are atomic orbitals.

Adding two atomic orbitals corresponds to constructive interference between two waves, thus reinforcing their intensity; the internuclear electron probability density is increased. The molecular orbital corresponding to the sum of the two H 1s orbitals is called a \(\sigma_{1s}\) combination (pronounced “sigma one ess”) (part (a) and part (b) in Figure \(\PageIndex{1}\)). In a sigma (\(\sigma\)) orbital, A bonding molecular orbital in which the electron density along the internuclear axis and between the nuclei has cylindrical symmetry; that is, all cross-sections perpendicular to the internuclear axis are circles. The subscript 1s denotes the atomic orbitals from which the molecular orbital was derived: The \(\approx\) sign is used rather than an = sign because we are ignoring certain constants that are not important to our argument.

Conversely, subtracting one atomic orbital from another corresponds to destructive interference between two waves, which reduces their intensity and causes a decrease in the internuclear electron probability density (part (c) and part (d) in Figure \(\PageIndex{1}\)). The resulting pattern contains a node where the electron density is zero. The molecular

\[
\text{σ} \approx 1s(A) + 1s(B)
\]

Figure \(\PageIndex{1}\): Molecular Orbitals for the H\(_2\) Molecule. (a) This diagram shows the formation of a bonding \(\sigma_{1s}\) molecular orbital for H\(_2\) as the sum of the wave functions (\(\Psi\)) of two H 1s atomic orbitals. (b) This plot of the square of the wave function (\(\Psi^2\)) for the bonding \(\sigma_{1s}\) molecular orbital illustrates the increased electron probability density between the two hydrogen nuclei. (Recall that the probability density is proportional to the square of the wave function.) (c) This diagram shows the formation of an antibonding \(\left\langle \sigma\right\rangle_{1s}^{(*)}\) molecular orbital for H\(_2\) as the difference of the wave functions (\(\Psi\)) of two H 1s atomic orbitals. (d) This plot of the square of the wave function (\(\Psi^2\)) for the \(\left\langle \sigma\right\rangle_{1s}^{(*)}\) antibonding molecular orbital illustrates the node corresponding to zero electron probability density between the two hydrogen nuclei.

\[
\left\langle \sigma\right\rangle_{1s} \approx 1s(A) - 1s(B)
\]
orbital corresponding to the difference is called \( \sigma_{1s}^{*} \) ("sigma one ess star"). In a sigma star (\( \sigma^{*} \)) orbital, an antibonding molecular orbital in which there is a region of zero electron probability (a nodal plane) perpendicular to the internuclear axis, there is a region of zero electron probability, a nodal plane, perpendicular to the internuclear axis:

\[
\sigma_{1s}^{*} \approx 1s(A) - 1s(B) \quad \text{(Eq3)}
\]

A molecule must have as many molecular orbitals as there are atomic orbitals. The electron density in the \( \sigma_{1s} \) molecular orbital is greatest between the two positively charged nuclei, and the resulting electron–nucleus electrostatic attractions reduce repulsions between the nuclei. Thus the \( \sigma_{1s} \) orbital represents a bonding molecular orbital. A molecular orbital that forms when atomic orbitals or orbital lobes with the same sign interact to give increased electron probability between the nuclei due to constructive reinforcement of the wave functions. In contrast, electrons in the \( \sigma_{1s}^{*} \) orbital are generally found in the space outside the internuclear region. Because this allows the positively charged nuclei to repel one another, the \( \sigma_{1s}^{*} \) orbital is an antibonding molecular orbital (a molecular orbital that forms when atomic orbitals or orbital lobes of opposite sign interact to give decreased electron probability between the nuclei due to destructive reinforcement of the wave functions).

Antibonding orbitals contain a node perpendicular to the internuclear axis; bonding orbitals do not.

---

Energy-Level Diagrams

Because electrons in the \( \sigma_{1s} \) orbital interact simultaneously with both nuclei, they have a lower energy than electrons that interact with only one nucleus. This means that the \( \sigma_{1s} \) molecular orbital has a lower energy than either of the hydrogen 1s atomic orbitals. Conversely, electrons in the \( \sigma_{1s}^{*} \) orbital interact with only one hydrogen nucleus at a time. In addition, they are farther away from the nucleus than they were in the parent hydrogen 1s atomic orbitals. Consequently, the \( \sigma_{1s}^{*} \) orbital has a higher energy than either of the hydrogen 1s atomic orbitals. The \( \sigma_{1s} \) (bonding) molecular orbital is stabilized relative to the 1s atomic orbitals, and the \( \sigma_{1s}^{*} \) (antibonding) molecular orbital is destabilized. The relative energy levels of these orbitals are shown in the energy-level diagram (a schematic drawing that compares the energies of the molecular orbitals (bonding, antibonding, and nonbonding) with the energies of the parent atomic orbitals) in Figure \( \text{Figure (3)} \)
The two available electrons (one from each H atom) in this diagram fill the bonding $\sigma_{1s}$ molecular orbital. Because the energy of the $\sigma_{1s}$ molecular orbital is lower than that of the two H 1s atomic orbitals, the H$_2$ molecule is more stable (at a lower energy) than the two isolated H atoms.

A bonding molecular orbital is \textit{always} lower in energy (more stable) than the component atomic orbitals, whereas an antibonding molecular orbital is \textit{always} higher in energy (less stable).

To describe the bonding in a homonuclear diatomic molecule (a molecule that consists of two atoms of the same element) such as H$_2$, we use molecular orbitals; that is, for a molecule in which two identical atoms interact, we insert the total number of valence electrons into the energy-level diagram (Figure \ref{PagelIndex2}). We fill the orbitals according to the \textit{Pauli principle} and \textit{Hund’s rule}: each orbital can accommodate a maximum of two electrons with opposite spins, and the orbitals are filled in order of increasing energy. Because each H atom contributes one valence electron, the resulting two electrons are exactly enough to fill the $\sigma_{1s}$ bonding molecular orbital. The two electrons enter an orbital whose energy is lower than that of the parent atomic orbitals, so the H$_2$ molecule is more stable than the two isolated hydrogen atoms. Thus molecular orbital theory correctly predicts that H$_2$ is a stable molecule. Because bonds form when electrons are concentrated in the space between nuclei, this approach is also consistent with our earlier discussion of electron-pair bonds.

**Bond Order in Molecular Orbital Theory**

In the Lewis electron structures, the number of electron pairs holding two atoms together was called the \textit{bond order}. In the molecular orbital approach, bond order \textit{One-half} the net number of bonding electrons in a molecule. is defined as \textit{one-half} the \textit{net} number of bonding electrons:

\[
\text{bond order} = \frac{\text{number of bonding electrons} - \text{number of antibonding electrons}}{2} \label{Eq4}
\]

To calculate the bond order of H$_2$, we see from Figure \ref{PagelIndex2} that the $\sigma_{1s}$ (bonding) molecular orbital contains two electrons, while the \$\sigma_{1s}^*\$ (antibonding) molecular orbital is empty. The bond order of H$_2$ is therefore
This result corresponds to the single covalent bond predicted by Lewis dot symbols. Thus molecular orbital theory and the Lewis electron-pair approach agree that a single bond containing two electrons has a bond order of 1. Double and triple bonds contain four or six electrons, respectively, and correspond to bond orders of 2 and 3.

We can use energy-level diagrams such as the one in Figure (PageIndex{2}) to describe the bonding in other pairs of atoms and ions where \( n = 1 \), such as the \( \text{H}_2^+ \) ion, the \( \text{He}_2^+ \) ion, and the \( \text{He}_2 \) molecule. Again, we fill the lowest-energy molecular orbitals first while being sure not to violate the Pauli principle or Hund’s rule.

Figure (PageIndex{3}): Molecular Orbital Energy-Level Diagrams for Diatomic Molecules with Only 1s Atomic Orbitals. (a) The \( \text{H}_2^+ \) ion, (b) the \( \text{He}_2^+ \) ion, and (c) the \( \text{He}_2 \) molecule are shown here.

Part (a) in Figure (PageIndex{3}) shows the energy-level diagram for the \( \text{H}_2^+ \) ion, which contains two protons and only one electron. The single electron occupies the \( \sigma_{1s} \) bonding molecular orbital, giving a \( \left( \sigma_{1s} \right)^1 \) electron configuration. The number of electrons in an orbital is indicated by a superscript. In this case, the bond order is \( (1-0)/2 = 1/2 \) because the bond order is greater than zero, the \( \text{H}_2^+ \) ion should be more stable than an isolated H atom and a proton. We can therefore use a molecular orbital energy-level diagram and the calculated bond order to predict the relative stability of species such as \( \text{H}_2^+ \). With a bond order of only 1/2 the bond in \( \text{H}_2^+ \) should be weaker than in the \( \text{H}_2 \) molecule, and the H–H bond should be longer. As shown in Table (PageIndex{1}), these predictions agree with the experimental data.

Part (b) in Figure (PageIndex{3}) is the molecular orbital energy-level diagram for \( \text{He}_2^+ \). This ion has a total of three valence electrons. Because the first two electrons completely fill the \( \sigma_{1s} \) molecular orbital, the Pauli principle states that the third electron must be in the \( \left( \sigma_{1s} \right)^2 \left( \sigma_{1s}^* \right)^1 \) electron configuration. This electron configuration gives a bond order of \( (2-1)/2 = 1/2 \). As with \( \text{H}_2^+ \), the \( \text{He}_2^+ \) ion should be stable, but the He–He bond should be weaker and longer than in \( \text{H}_2 \). In fact, the \( \text{He}_2^+ \) ion can be prepared, and its properties are consistent with our predictions (Table (PageIndex{1})).

**Table (PageIndex{1}): Molecular Orbital Electron Configurations, Bond Orders, Bond Lengths, and Bond Energies for some Simple Homonuclear Diatomic Molecules and Ions**

<table>
<thead>
<tr>
<th>Molecule or Ion</th>
<th>Electron Configuration</th>
<th>Bond Order</th>
<th>Bond Length (pm)</th>
<th>Bond Energy (kJ/mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{H}_2^+ )</td>
<td>( \left( \sigma_{1s} \right)^1 )</td>
<td>1/2</td>
<td>106</td>
<td>269</td>
</tr>
<tr>
<td>( \text{H}_2 )</td>
<td>( \left( \sigma_{1s} \right)^2 )</td>
<td>1</td>
<td>74</td>
<td>436</td>
</tr>
<tr>
<td>( \text{He}_2^+ )</td>
<td>( \left( \sigma_{1s} \right)^2 \left( \sigma_{1s}^* \right)^1 )</td>
<td>1/2</td>
<td>108</td>
<td>251</td>
</tr>
</tbody>
</table>
Finally, we examine the He$_2$ molecule, formed from two He atoms with $1s^2$ electron configurations. Part (c) in Figure (PageIndex{3}) is the molecular orbital energy-level diagram for He$_2$. With a total of four valence electrons, both the $\sigma$$_{1s}$ bonding and $\sigma$$_{1s}^*$ antibonding orbitals must contain two electrons. This gives a $\left(\sigma$$_{1s}\right)^2\left(\sigma$$_{1s}^*\right)^1$ electron configuration, with a predicted bond order of $(2 - 2) ÷ 2 = 0$, which indicates that the He$_2$ molecule has no net bond and is not a stable species. Experiments show that the He$_2$ molecule is actually less stable than two isolated He atoms due to unfavorable electron–electron and nucleus–nucleus interactions.

In molecular orbital theory, electrons in antibonding orbitals effectively cancel the stabilization resulting from electrons in bonding orbitals. Consequently, any system that has equal numbers of bonding and antibonding electrons will have a bond order of 0, and it is predicted to be unstable and therefore not to exist in nature. In contrast to Lewis electron structures and the valence bond approach, molecular orbital theory is able to accommodate systems with an odd number of electrons, such as the H$_2^+$ ion.

Note

In contrast to Lewis electron structures and the valence bond approach, molecular orbital theory can accommodate systems with an odd number of electrons.

Example \(\PageIndex{1}\)

Use a molecular orbital energy-level diagram, such as those in Figure (PageIndex{3}), to predict the bond order in the He$_2^{2+}$ ion. Is this a stable species?

**Given:** chemical species

**Asked for:** molecular orbital energy-level diagram, bond order, and stability

**Strategy:**

A. Combine the two He valence atomic orbitals to produce bonding and antibonding molecular orbitals. Draw the molecular orbital energy-level diagram for the system.

B. Determine the total number of valence electrons in the He$_2^{2+}$ ion. Fill the molecular orbitals in the energy-level diagram beginning with the orbital with the lowest energy. Be sure to obey the Pauli principle and Hund’s rule while doing so.

C. Calculate the bond order and predict whether the species is stable.
Solution:

A Two He 1s atomic orbitals combine to give two molecular orbitals: a $\sigma_{1s}$ bonding orbital at lower energy than the atomic orbitals and a $\left(\sigma_{1s}\right)^*\left(\sigma_{1s}\right)$ antibonding orbital at higher energy. The bonding in any diatomic molecule with two He atoms can be described using the following molecular orbital diagram:

![Molecular orbital diagram](image)

B The He$_2^{2+}$ ion has only two valence electrons (two from each He atom minus two for the +2 charge). We can also view He$_2^{2+}$ as being formed from two He$^+$ ions, each of which has a single valence electron in the 1s atomic orbital. We can now fill the molecular orbital diagram:

![Molecular orbital diagram](image)

The two electrons occupy the lowest-energy molecular orbital, which is the bonding ($\sigma_{1s}$) orbital, giving a $(\sigma_{1s})^2$ electron configuration. To avoid violating the Pauli principle, the electron spins must be paired. C So the bond order is

$$\left(\frac{2-0}{2}\right) = 1$$

He$_2^{2+}$ is therefore predicted to contain a single He–He bond. Thus it should be a stable species.

Exercise $\PageIndex{1}$

Use a molecular orbital energy-level diagram to predict the valence-electron configuration and bond order of the H$_2^{2-}$ ion. Is this a stable species?

Answer:

H$_2^{2-}$ has a valence electron configuration of $\left(\sigma_{1s}\right)^2\left(\sigma_{1s}\right)^*\left(\sigma_{1s}\right)^2$ with a bond order of 0. It is therefore predicted to be unstable.

So far, our discussion of molecular orbitals has been confined to the interaction of valence orbitals, which tend to lie farthest from the nucleus. When two atoms are close enough for their valence orbitals to overlap significantly, the filled inner electron shells are largely unperturbed; hence they do not need to be considered in a molecular orbital scheme. Also, when the inner orbitals are completely filled, they contain exactly enough electrons to completely fill both the bonding and antibonding molecular orbitals that arise from their interaction. Thus the interaction of filled shells always gives a bond order of 0, so filled shells are not a factor when predicting the stability of a species. This means that we can focus our
attention on the molecular orbitals derived from valence atomic orbitals.

A molecular orbital diagram that can be applied to any homonuclear diatomic molecule with two identical alkali metal atoms (Li\(_2\) and Cs\(_2\), for example) is shown in part (a) in Figure \(\PageIndex{4}\), where M represents the metal atom. Only two energy levels are important for describing the valence electron molecular orbitals of these species: a \(\sigma_{ns}\) bonding molecular orbital and a \(\sigma^*_{ns}\) antibonding molecular orbital. Because each alkali metal (M) has an \(ns^1\) valence electron configuration, the M\(_2\) molecule has two valence electrons that fill the \(\sigma_{ns}\) bonding orbital. As a result, a bond order of 1 is predicted for all homonuclear diatomic species formed from the alkali metals (Li\(_2\), Na\(_2\), K\(_2\), Rb\(_2\), and Cs\(_2\)). The general features of these M\(_2\) diagrams are identical to the diagram for the H\(_2\) molecule in Figure \(\PageIndex{2}\). Experimentally, all are found to be stable in the gas phase, and some are even stable in solution.

Similarly, the molecular orbital diagrams for homonuclear diatomic compounds of the alkaline earth metals (such as Be\(_2\)), in which each metal atom has an \(ns^2\) valence electron configuration, resemble the diagram for the He\(_2\) molecule in part (c) in Figure \(\PageIndex{3}\). As shown in part (b) in Figure \(\PageIndex{4}\), this is indeed the case. All the homonuclear alkaline earth diatomic molecules have four valence electrons, which fill both the \(\sigma_{ns}\) bonding and the \(\sigma^*_{ns}\) (nonbonding) levels, leading to a predicted bond order of 0. Thus Be\(_2\), Mg\(_2\), Ca\(_2\), Sr\(_2\), and Ba\(_2\) are all expected to be unstable, in agreement with experimental data. In the solid state, however, all the alkali metals and the alkaline earth metals exist as extended lattices held together by metallic bonding. At low temperatures, \((\text{Be}_2)\) is stable.

Example \(\PageIndex{2}\)

Use a qualitative molecular orbital energy-level diagram to predict the valence electron configuration, bond order, and likely existence of the Na\(_2^-\) ion.

**Given:** chemical species

**Asked for:** molecular orbital energy-level diagram, valence electron configuration, bond order, and stability

**Strategy:**

A. Combine the two sodium valence atomic orbitals to produce bonding and antibonding molecular orbitals. Draw the molecular orbital energy-level diagram for this system.

B. Determine the total number of valence electrons in the Na\(_2^-\) ion. Fill the molecular orbitals in the energy-level diagram beginning with the orbital with the lowest energy. Be sure to obey the Pauli principle and Hund’s rule while...
doing so.

C. Calculate the bond order and predict whether the species is stable.

Solution:

A Because sodium has a \([\text{Ne}]3s^1\) electron configuration, the molecular orbital energy-level diagram is qualitatively identical to the diagram for the interaction of two 1s atomic orbitals.

B The Na\(_2^-\) ion has a total of three valence electrons (one from each Na atom and one for the negative charge), resulting in a filled \(\sigma_{3s}\) molecular orbital, a half-filled \(\sigma_{3s}^*\) and a \(\left(\sigma_{3s}\right)^2\left(\sigma_{3s}^*\right)^1\) electron configuration.

\[
\begin{array}{c}
\text{Na (AO)} & \text{Na}\(_2^-\) (MOs) & \text{Na (AO)} \\
\sigma_{3s} & 3s & \sigma_{3s} \\
\end{array}
\]

C The bond order is \(2-1\div2=1/2\) With a fractional bond order, we predict that the Na\(_2^-\) ion exists but is highly reactive.

Exercise \(\PageIndex{2}\)

Use a qualitative molecular orbital energy-level diagram to predict the valence electron configuration, bond order, and likely existence of the Ca\(_2^+\) ion.

**Answer:** Ca\(_2^+\) has a \(\left(\sigma_{4s}\right)^2\left(\sigma_{4s}^*\right)^1\) electron configurations and a bond order of 1/2 and should exist.

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