Dimensional analysis is amongst the most valuable tools physical scientists use. Simply put, it is the conversion between an amount in one unit to the corresponding amount in a desired unit using various conversion factors. This is valuable because certain measurements are more accurate or easier to find than others.

Performing Dimensional Analysis

The use of units in a calculation to ensure that we obtain the final proper units is called dimensional analysis. For example, if we observe experimentally that an object’s potential energy is related to its mass, its height from the ground, and to a gravitational force, then when multiplied, the units of mass, height, and the force of gravity must give us units corresponding to those of energy.

Energy is typically measured in joules, calories, or electron volts (eV), defined by the following expressions:

- $1 \text{ J} = 1 \left( \text{kg} \cdot \text{m}^2 \right) / \text{s}^2 = 1 \text{ coulomb-volt}$
- $1 \text{ cal} = 4.184 \text{ J}$
- $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$

Performing dimensional analysis begins with finding the appropriate conversion factors. Then, you simply multiply the values together such that the units cancel by having equal units in the numerator and the denominator. To understand this process, let us walk through a few examples.

Example \(\PageIndex{1}\)

Imagine that a chemist wants to measure out 0.214 mL of benzene, but lacks the equipment to accurately measure such a small volume. The chemist, however, is equipped with an analytical balance capable of measuring to ±0.0001 g. Looking in a reference table, the chemist learns the density of benzene ($\rho = 0.8765 \text{ g/mL}$). How many grams of benzene should the chemist use?

**Solution**

\[
0.214 \; \text{cancel(mL)} \left( \dfrac{0.8765 \; \text{g}}{\text{1 cancel(mL)}} \right) = 0.187571 \; \text{g}
\]

Notice that the mL are being divided by mL, an equivalent unit. We can cancel these our, which results with the 0.187571 g. However, this is not our final answer, since this result has too many significant figures and must be rounded down to three significant digits. This is because 0.214 mL has three significant digits and the conversion factor had four significant digits. Since 5 is greater than or equal to 5, we must round the preceding 7 up to 8.

Hence, the chemist should weigh out 0.188 g of benzene to have 0.214 mL of benzene.

Example \(\PageIndex{2}\)

To illustrate the use of dimensional analysis to solve energy problems, let us calculate the kinetic energy in joules of a 320 g object traveling at 123 cm/s.
Solution

To obtain an answer in joules, we must convert grams to kilograms and centimeters to meters. Using Equation 5.4, the calculation may be set up as follows:

\[
KE = \frac{1}{2}mv^2 = \frac{1}{2}(g) \left(\frac{kg}{g}\right) \left(\frac{m}{cm}\right)^2
\]

\[
= (g)\left(\frac{kg}{g}\right) \left(\frac{m^2}{s^2}\right) \left(\frac{m^2}{cm^2}\right) = \frac{kg \cdot m^2}{s^2}
\]

\[
= \frac{1}{2} \times 320 \; \cancel{g} \left( \frac{1\; kg}{1000 \; \cancel{g}} \right) \left(\frac{123 \; \cancel{cm}}{1 \; s}\right)^2 = \frac{0.320 \; kg}{2} \left(\frac{123 \; m}{s(100)}\right)^2
\]

\[
= \frac{1}{2} 0.320 \; kg \left[ \frac{(123)^2 \; m^2}{s^2(100)^2} \right] = 0.242 \; \frac{kg \cdot m^2}{s^2} = 0.242 \; J
\]

Alternatively, the conversions may be carried out in a stepwise manner:

Step 1: convert (g) to (kg)

\[
320 \; \cancel{g} \left( \frac{1\; kg}{1000 \; \cancel{g}} \right) = 0.320 \; kg
\]

Step 2: convert (cm) to (m)

\[
123 \; \cancel{cm} \left( \frac{1 \; m}{100 \; \cancel{cm}} \right) = 1.23 \; m
\]

Now the natural units for calculating joules is used to get final results

\[
KE = \frac{1}{2} 0.320 \; kg \left(1.23 \; ms\right)^2 = \frac{1}{2} 0.320 \; kg \left(1.513 \; \frac{m^2}{s^2}\right) = 0.242 \; \frac{kg \cdot m^2}{s^2} = 0.242 \; J
\]

Of course, steps 1 and 2 can be done in the opposite order with no effect on the final results. However, this second method involves an additional step.

Example \ref{PageIndex3})

Now suppose you wish to report the number of kilocalories of energy contained in a 7.00 oz piece of chocolate in units of kilojoules per gram.

Solution

To obtain an answer in kilojoules, we must convert 7.00 oz to grams and kilocalories to kilojoules. Food reported to contain a value in Calories actually contains that same value in kilocalories. If the chocolate wrapper lists the caloric content as 120 Calories, the chocolate contains 120 kcal of energy. If we choose to use multiple steps to obtain our answer, we can begin with the conversion of kilocalories to kilojoules:

\[
120 \; \cancel{kcal} \left( \frac{1000 \; \cancel{cal}}{\cancel{kcal}} \right) \left( \frac{4.184 \; \cancel{J}}{\cancel{cal}} \right) = 502 \; kJ
\]

We next convert the 7.00 oz of chocolate to grams:
The number of kilojoules per gram is therefore

\[
\frac{502 \; \text{kJ}}{199 \; \text{g}} = 2.52 \; \text{kJ/g}
\]

Alternatively, we could solve the problem in one step with all the conversions included:

\[
\left(\frac{120 \; \text{cal}}{7.00 \; \text{oz}}\right) \left(\frac{1000 \; \text{cal}}{1 \; \text{kcal}}\right) \left(\frac{4.184 \; \text{J}}{1 \; \text{cal}}\right) \left(\frac{1 \; \text{kJ}}{1000 \; \text{J}}\right) \left(\frac{1 \; \text{oz}}{28.35 \; \text{g}}\right) = 2.53 \; \text{kJ/g}
\]

The discrepancy between the two answers is attributable to rounding to the correct number of significant figures for each step when carrying out the calculation in a stepwise manner. Recall that all digits in the calculator should be carried forward when carrying out a calculation using multiple steps. In this problem, we first converted kilocalories to kilojoules and then converted ounces to grams. Skill Builder ES2 allows you to practice making multiple conversions between units in a single step.

**Problems**

1. Write a single equation to show how to convert
   a. \(\text{cm/min}\) to \(\text{km/h}\);
   b. \(\text{cal/oz}\) to \(\text{J/g}\);
   c. \(\text{lb/in}^2\) to \(\text{kg/m}^2\) and
   d. \(\text{°C/s}\) to \(\text{K/h}\).

2. How many Calories are contained in an 8.0 oz serving of green beans if their fuel value is 1.5 kJ/g?

3. Gasoline has a fuel value of 48 kJ/g. How much energy in joules can be obtained by filling an automobile's 16.3 gal tank with gasoline, assuming gasoline has a density of 0.70 g/mL?

**Solutions**

1. Converting from one compound unit to another
   a. \(\left(\frac{\text{cm}}{\text{min}}\right) \left(\frac{1 \; \text{m}}{100 \; \text{cm}}\right) \left(\frac{1 \; \text{km}}{1000 \; \text{m}}\right) \left(\frac{60 \; \text{min}}{1 \; \text{h}}\right) = \text{km/h}\)
   b. \(\left(\frac{\text{cal}}{\text{oz}}\right) \left(\frac{4.184 \; \text{J}}{1 \; \text{cal}}\right) \left(\frac{16 \; \text{oz}}{1 \; \text{lb}}\right) \left(\frac{1 \; \text{lb}}{453.59 \; \text{g}}\right) = \text{J/g}\)
   c. \(\left(\frac{\text{lb}}{\text{in}^2}\right) \left(\frac{16 \; \text{oz}}{1 \; \text{lb}}\right) \left(\frac{28.35 \; \text{g}}{1 \; \text{oz}}\right) \left(\frac{1 \; \text{kg}}{1000 \; \text{g}}\right) \left(\frac{36 \; \text{in.}^2}{1 \; \text{yd}^2}\right) = \text{kg/m}^2\)
   d. \(\left(\frac{\text{°C}}{\text{s}}\right) \left(\frac{60 \; \text{s}}{1 \; \text{min}}\right) + 273.15 = \text{K/h}\)

2. Our goal is to convert 1.5 kJ/g to Calories in 8 oz:\(\left(\frac{1.5 \; \text{kJ}}{1 \; \text{g}}\right) \left(\frac{1000 \; \text{J}}{1 \; \text{kJ}}\right) \left(\frac{1 \; \text{cal}}{4.184 \; \text{J}}\right) \left(\frac{1 \; \text{oz}}{28.35 \; \text{g}}\right) = 3.5 \; \text{cal/oz}\)
\[
\text{3. Our goal is to use the energy content, 48 kJ/g, and the density, 0.70 g/mL, to obtain the number of joules in 16.3 gal of gasoline: }
\left(\dfrac{48 \; \cancel{kJ}}{g}\right)\left(\dfrac{1000 \; J}{\cancel{kJ}}\right)\left(\dfrac{0.70 \; \cancel{g}}{\cancel{mL}}\right)\left(\dfrac{1000 \; \cancel{mL}}{\cancel{L}}\right)\left(\dfrac{3.79 \; \cancel{L}}{\cancel{gal}}\right)\left(16.3 \; \cancel{gal}\right) = 2.1 \times 10^9 \; J
\]