A Further Word on Unpaired and Paired Data

Distinguishing between unpaired and paired data always seems to create confusion. A good first approach to deciding if you have unpaired or paired data is to look for some key words in the problem. For example, a problem stating that a single sample is analyzed multiple times by two methods or two analysts is almost certainly an example of unpaired data. On the other hand, a problem in which many samples are each analyzed by two methods or two analysts *might* be an example of paired data. You'll note that I used the word might. The key is to determine if an individual sample is analyzed by both methods or both analysts, or if every analysis is done on a different sample. Take the penny problem as an example. If we gather 12 different pennies and each penny is included in data set 1 and data set 2, then the data are paired. If we gather 12 pennies and analyze 6 for data set 1 and 6 for data set 2, then the data are unpaired. Another way to make this distinction is to ask the following question - Is entry X in data set 1 tied uniquely to entry X in data set 2. If the answer is "yes", then the data must be paired.

Why is this distinction important? The reason is that a *t*-test is limited by the standard deviation. To distinguish between the mean values for two methods, for example, we want the standard deviation (either s_{pool}, or s_1 and s_2) to reflect only the precisions of the methods being compared. If there is another source of variability, such as a significant variation between the samples, then neither a pooled standard deviation nor the individual standard deviations for the two data sets will accurately reflect the precisions of the two methods. Consider the set of data shown on the right in which the results for Set B are always greater than the results for Set A. If you treat the data as unpaired you obtain mean values of 72.92 mg and 75.92 mg, and standard deviations of 11.57 and 11.74 for Set A and Set B, respectively. An unpaired *t*-test shows that there is no significant difference between the two sets. This should strike you as strange; a difference of 3 mg/penny seems significant. The problem is that the standard deviations of 11.57 and 11.74 include both the precision of the methods used to collect the data in Sets A and B, **and** the variability in Cu between pennies (which is clearly quite substantial). If you treat this as paired data, the average difference of 3.00 mg and the standard deviation of 2.30 leads to the conclusion that the data sets are indeed different at $\alpha = 0.05$.

	Cu (mg/penny)	
Penny Number	Set A	Set B
1	97	102
2	71	73
3	63	68
4	75	76
5	65	66
6	62	69
7	72	72
8	66	69
9	91	96
10	76	77
11	78	79
12	59	64