

Worksheet 3A Solutions

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October 15, 2016

1. Suppose

$$\hat{A} = \frac{d}{dx} \tag{1}$$

(a) What is the expression for $\hat{A}f(x)$?

$$\hat{A}f(x) = \frac{df(x)}{dx}$$

(b) If $f(x) = \frac{1}{x^2}$, what is $\hat{A}f(x)$?

$$\hat{A}f(x) = -\frac{2}{x}$$

(c) What is an example of \hat{A} that's linear?

Derivatives, integral, multiplication by a factor etc.

(d) What is an example of \hat{A} that's not linear?

$\cos()$, $\log()$, $f(x) = x + c$, etc

2. An eigenvalue equation takes the form:

$$\hat{A} = af(x) \tag{2}$$

Suppose $\hat{A} = \frac{d}{dx}$ and $f(x) = \exp(6x)$.

(a) What is the eigenvalue of \hat{A} operating on $f(x)$?

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(b) For $\hat{A} = \frac{d}{dx}$, can any mathematical function, $g(x)$ serve as the eigenfunction of \hat{A} ? Give examples that would not work.

No, some exceptions include $\sin()$, $\cos()$, $\log()$.

(c) Suppose $\hat{B} = \frac{d^2}{dx^2}$. What function could be an eigenfunction of \hat{B} ? What is the corresponding eigenvalue?

$\sin(kx)$, where the eigenvalue is $-k$

3. Suppose that $\hat{A} = \frac{d}{dx}$ and $\hat{B} = x^2$

(a) For any function, $f(x)$, what is $\hat{A}f(x)$?

$$\hat{A}f(x) = \frac{df(x)}{dx}$$

(b) What is $\hat{B}f(x)$?

$$\hat{B}f(x) = x^2 f(x)$$

(c) What is $\hat{A}\hat{B}f(x)$?

$$\hat{A}\hat{B}f(x) = x^2 \frac{df(x)}{dx} + 2xf(x)$$

(d) What is $\hat{B}\hat{A}f(x)$?

$$\hat{B}\hat{A}f(x) = x^2 \frac{df(x)}{dx}$$

(e) Is $\hat{A}\hat{B}f(x) = \hat{B}\hat{A}f(x)$? Why?

They are not equal because $x^2 \frac{df(x)}{dx} + 2xf(x) \neq x^2 \frac{df(x)}{dx}$ and therefore, they do not commute.

4. Suppose that $\hat{A} = \frac{d}{dx}$ and $\hat{B} = 10$

(a) What is $\hat{A}f(x)$?

$$\hat{A}f(x) = \frac{df(x)}{dx}$$

(b) What is $\hat{B}f(x)$?

$$\hat{B}f(x) = 10f(x)$$

(c) What is $\hat{A}\hat{B}f(x)$?

$$\hat{A}\hat{B}f(x) = 10 \frac{df(x)}{dx}$$

(d) What is $\hat{B}\hat{A}f(x)$?

$$\hat{B}\hat{A}f(x) = 10 \frac{df(x)}{dx}$$

(e) Do \hat{A} and \hat{B} commute?

Yes they do.