Scientific Notation & Logarithms

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Introduction: Convention of Expressing Any Base 10 Number As a Product of a Number Between One and 9, multiplied by 10 to the Power of Some Exponent.

Exponentiation is the replication of multiplication the way multiplication is the replication of addition

$10^0 = 1$
$10^1 = 10$ $10^{-1} = 1/10 = 0.1$
$10^2 = 10(10) = 100$ $10^{-2} = 1/[10(10)] = 1/100 = 0.01$
$10^3 = 10(10)(10) = 1000$ $10^{-3} = 1/[10(10)(10)] = 1/1000 = 0.001$

Advantages of Scientific Notation:

1. Allows Awkwardly Large and Small Numbers to Be Expressed in Term of Compact and Easily Written Numbers

2. Allows Accurate Representation of the Number of Significant Figures in a Number, That Is a Measurement's Precision, the "Certainty" of Our Measurements

Convert the following numbers to scientific notation
            0.00456 and 456.00

A) 0.00456   (A number smaller than 1)
        Step 1: Since this number is less than 1 you need identify the power of 10 which you can multiply it by and set one digit to the left of the decimal. Here, we can multiply by 1000 or 10^3 and force the original number to have one digit to the left of the decimal, but we have changed its value.
0.00456 x 10^3 = 4.56   (note, which is not equal to the original number)

        Step 2: Since we multiplied it by 10^3 we must also divide by 10^3 (effectively multiplying the original number by 1 and not changing it's value). Than we express the factor in the denominator as a power of 10.

0.00456

$$= 0.00456 x \frac{10^3}{10^3}$$

$$= \frac{0.00456 x 10^3}{10^3}$$

$$= \frac{4.56}{10^3}$$

$$= 4.56 x 10^{-3}$$

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Note, once you get the hang of this you can just count the number of digits you need to move the decimal to bring the original number to one digit to the left of the decimal and multiply that number by 10 to the negative value of those digits. Here we moved the decimal three positions to the right, so we multiply by 10^{-3} (ie., divide by 10^{3}).

B. 456.00, (A number larger than 10)
Since this number has a value greater than 9 we must divide it by a factor of 10 to make the first non-zero digit value between 1 and 9.

456.00

$$= 456.00 \frac{10^2}{10^2}$$

$$= \frac{456.00}{10^2} x 10^2$$

$$= 4.56 x 10^2$$

Note, once you get the hang of this you can just count the number of places you move the decimal to the right (resulting in a value with one digit to the left of the decimal) and then multiply by 10 to the power of that number (2 in this case).

Using Scientific Notation to Express Significant Digits:

How can you express the number 400 to 2 significant digits?
You must use scientific notation.
4.0×10^2

As scientific notation always has a decimal point there is never a problem expressing significant digits.

Problem Set 1:

Convert the following measured values to scientific notation and express answers to the proper number of significant figures.

a. 0.00045602	b. 50444000	c. 50444000.
d. 32.010	e. 253	f. 0.0034

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Multiplication of numbers in Scientific notation

Try plugging the following number into your calculator. Odds are you will get an error message.

$$\frac{\left(9.47x10^{598}\right)\left(7.39x10^{-98}\right)}{\left(1.432x10^{-645}\right)\left(2.46x10^{4}\right)}$$

You will often get numbers which are bigger or smaller than a calculator can handle. You need to treat the power separately from the number, so break into two terms.

$$\frac{\left(9.47x10^{598}\right)\left(7.39x10^{-98}\right)}{\left(1.432x10^{-645}\right)\left(2.46x10^{4}\right)}$$

$$=\left(\frac{9.47x7.39}{1.432x2.46}\right)\left(\frac{10^{598}x10^{-98}}{10^{-645}x10^{4}}\right)=$$

$$=19.866x10^{(598-98+645-4)}$$

$$=19.9x10^{1141}=1.99x10^{1142}$$

Addition and Subtraction of Numbers in Scientific Notation

Consider the following problem
$4.860 \times 10^{12} + 9.7 \times 10^{10} + 3.68x10^{11}$

You first need to express all numbers to the same power so you can line up the decimal point. It is suggested that you choose the largest power and make everything else a fraction.

$$\begin{array}{r} 4.860x10^{12} \\ +0.097x10^{12} \\ \underline{+0.368x10^{12}} \\ 5.325x10^{12} \end{array}$$

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Problem Set 2: Solve the following problems

1. $3.03860 \times 10^{-32} + 3.7 \times 10^{-30} + 3.68 \times 10^{-31}$

2. $3.03860 \times 10^{-32} - 3.7 \times 10^{-30} + 3.68 \times 10^{-31}$

3. $\dfrac{\left(9.7 x 10^{583}\right)\left(7.339 x 10^{98}\right)}{\left(5.432 x 10^{-645}\right)\left(3.446 x 10^{-4484}\right)}$

4. $\dfrac{\left(9.7 x 10^{583}\right)\left(7.339 x 10^{-98}\right)}{\left(5.432 x 10^{-645}\right)\left(3.446 x 10^{4484}\right)}$

5. $\dfrac{\left(9.700 x 10^{583}\right)\left(7.339 x 10^{98}\right)}{\left(5.432 x 10^{-645}\right)\left(3.446 x 10^{-4484}\right)}$

6. $\dfrac{\left(9.70 x 10^{-583}\right)\left(7.339 x 10^{-98}\right)}{\left(5.432 x 10^{645}\right)\left(3.446 x 10^{4484}\right)}$

7. $\dfrac{\left(9.70 x 10^{5} + 6.3 x 10^{4}\right)\left(3.33 x 10^{5}\right)}{\left(5.00 x 10^{45}\right)\left(3.20 x 10^{-5}\right)}$

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Logarithms (Base 10)

The log is the power you take a base to:

$10^0 = 1$	log=0		
$10^1 = 10$	log=1	$10^{-1} = 1/10 = 0.1$	log=-1
$10^2 = 10(10) = 100$	log=2	$10^{-2} = =0.01$	log=-2
$10^3=10(10)(10) = 1000$	log=3	$10^{-3} = 1/1000 = 0.001$	log=-3

NOTE: 10 to the power of a fraction is a number between 1 and 10.
($10^{0.01} = 1.023$,$10^{0.1} = 1.259$, $10^{0.5} = 3.16$, $10^{0.9} = 7.94$, $10^{0.99} = 9.77$)

The anti log is 10 to the power of a number
Now just as you add exponents, you add the log:

$10^2 + 10^4 = 10^6$, so log100 x log10,000 = 2+4=6 (and the antilog is 10^6)

The easiest way to understand logs is by comparing it to scientific notation, as the log of a number has two parts which are related to scientific notation.

$4537 = 4.537 \times 10^3 = 10^{0.6568} \times 10^3 = 10^{(0.6568 + 3)} = 10^{3.6568}$
Log $4537 = $ log4.537 x $\log 10^3 = 0.6568 + 3 = 3.3568$

That is a log has two parts, the characteristic and the mantissa. The characteristic is the power of 10 you multiply the first part of the log by and the mantissa is 10 to the power which gives the number between 1 and 10.

So for log $4537 = 3.3568$, 3 = characteristic and 0.3568 = mantissa
To represent in scientific notation

$$4537 = 10^{3.6568} = 10^{0.6568} \times 10^3 = 4.537 \times 10^3$$

Your calculator will perform these for you. In gen chem 2, you will need to use logs extensively and have mastery over the following relationships

$a^{-n} = \dfrac{1}{a^n}$	$\dfrac{1}{a^{-n}} = a^n$	$\ln xy = \ln x + \ln y$	$\log xy = \log x + \log y$
$a^m a^n = a^{(m+n)}$	$\dfrac{a^m}{a^n} = a^{(m-n)}$	$\ln \dfrac{x}{y} = \ln x - \ln y$	$\log \dfrac{x}{y} = \log x - \log y$
		$\ln x^k = k \ln x$	$\log x^k = k \log x$
$(ab)^k = a^k b^k$	$\left(\dfrac{a}{b}\right)^k = \dfrac{a^k}{b^k}$	$\ln \sqrt[k]{x} = \ln x^{\frac{1}{k}} = \dfrac{1}{k} \ln x$	$\log \sqrt[k]{x} = \log x^{\frac{1}{k}} = \dfrac{1}{k} logx$
$(a^m)^n = a^{mn}$	$(a^m)^{-n} = \dfrac{1}{a^{mn}}$	$\ln e^k = k$	$\log 10^k = k$
		$\ln x = 2.302585...\log x$	$\log x = 0.434294... \ln x$