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(Group Name)\_\_\_\_\_  
(Date)

## Exercise on Fractions and Isotopic Composition

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In this problem set you will be given questions pertaining to the isotopic composition of various elements. Use the following symbols for your algebraic equations

$m$  = mass;  $m_1$  = mass of species 1,  $m_2$  = mass of species 2.....  $m_{ave}$  = Average Mass

$X$  = fraction;  $X_1$  = fraction of species 1,  $X_2$  = Fraction of species 2...

**Problem 1: Consider an element which has 3 naturally occurring isotopes. The isotopic masses and percent abundances are: 53.9396 amu (5.845%), 55.9349 amu (91.754%) and 56.9354amu . What is the Average Mass?**

a. Define algebraic mass variables ( $m_1 =$  ,  $m_2 =$ .....) and write values for those given in the problem statement, use question marks for unknowns (there should be 4)

**$m_1 = 53.9396$  amu ,  $m_2 = 55.9349$  amu  $m_3 = 56.94$  amu ,  $m_{ave} = ?$**

b. Define fractional abundance variables ( $X_1 =$  ,  $X_2 =$ .....) and write values for those given in the problem statement, use question marks for unknowns (there should be 3)

**$X_1 = 0.05845$  ,  $X_2 = 0.91754$  ,  $X_3 = ?$**

c. Calculate value of remaining fractional abundance variable (think about the concept of a fraction and the fact that there are only three types of isotopes).

**$(X_3 = 1 - 0.05845 - 0.91754 = 0.02401)$**

d. Set up and algebraic equation and identify variable you are solving for. Solve equation for unknown.

**$m_1X_1 + m_2X_2 + m_3X_3 = m_{ave}$**

e. Substitute numerical values into algebraic equation and solve

$$m_{\text{ave}} = 0.05845(53.9396\text{amu}) + 0.91754(55.9349\text{amu}) + 0.02401(56.9354\text{amu}) = 55.84\text{amu}$$

f. What element is this? **Iron**

**Problem 2: Copper has two naturally occurring isotopes. One of the isotopes has an atomic weight of 62.9 amu and an abundance of 69.1%. What is the atomic weight (amu) of the other isotope?**

a. Define algebraic mass variables, write down givens and unknowns (there should be 3):

$$m_1 = 62.9\text{ amu}, m_2 = ?\text{ amu}, m_{\text{ave}} = ?$$

b. Define fractional abundance variables, write down givens and unknowns (there should be 2):

$$X_1 = 0.691, X_2 = ?$$

c. Set up (write down) algebraic equation based on the concept of fractional abundance of isotopes.

$$m_1 X_1 + m_2 X_2 = m_{\text{ave}}$$

d. Identify unknown you are solving for and rearrange algebraic equation to show solution for your desired unknown.

$$m_2 = \frac{m_{\text{ave}} - m_1 X_1}{X_2}$$

e. You can not arithmetically solve one equation if you have more than one unknown. You have three unknowns, and so you need to look somewhere else for information. What are the 2 additional unknowns?

$$m_{\text{ave}} \text{ and } X_2.$$

f. What is the average atomic weight? Remember, this is a chemistry class and you should be able to find that for any element. Write it down in terms of your algebraic variables.  $m_{\text{ave}} = 63.546\text{ amu}$

g. There are only two isotopes. How can you determine the fractional abundance of the unknown isotope? Write it down in terms of your algebraic variables.

$$X_2 = 1 - 0.691 = 0.309$$

h. Substitute numerical values into the algebraic equation and arithmetically solve for the atomic weight of the other unknown.

$$m_2 = \frac{63.546amu - 62.9amu(0.691)}{0.309} = 65.0amu$$

**Problem 3: Natural Chlorine consists of  $^{35}\text{Cl}$  (34.97 amu) and  $^{37}\text{Cl}$  (36.97 amu). What is the Percent abundance of the lighter isotope?**

a. Define algebraic mass variables, write down givens and unknowns (there should be 3):

$$m_1 = 34.97 \text{ amu}, m_2 = 36.97 \text{ amu}, m_{\text{ave}} = ?$$

b. Define fractional abundance variables, write down givens and unknowns (there should be 2):

$$X_1 = ?, X_2 = ?$$

c. Set up (write down) algebraic equation based on the concept of fractional abundance of isotopes.

$$m_1 X_1 + m_2 X_2 = m_{\text{ave}}$$

d. Identify unknown you are solving for and rearrange algebraic equation to show solution for your desired unknown.

$$X_1 = \frac{m_{\text{ave}} - m_2 X_2}{m_1}$$

e. You can not arithmetically solve one equation if you have more than one unknown. You have three unknowns, and so you need to look somewhere else for information. What are the 2 additional unknowns?

$$m_{\text{ave}} \text{ and } X_2.$$

f. What is the average atomic weight? Remember, this is a chemistry class and you should be able to find that for any element. Write it down in terms of your algebraic variables.  $m_{\text{ave}} = 35.45 \text{ amu}$

g. There are only two isotopes. How can you determine the fractional abundance of the additional unknown isotope (not the one are solving for)? Write it down in terms of you algebraic variables.

$$1 = X_1 + X_2 \text{ so } X_2 = 1 - X_1$$

h. Substitute this expression into the equation solving for the unknown you are after and algebraically solve for the unknown

$$m_{\text{ave}} = m_1 X_1 + m_2 X_2 \quad \text{so} \quad m_{\text{ave}} = m_1 X_1 + m_2 (1 - X_1)$$

$$X_1 = \frac{m_{\text{ave}} - m_2}{m_1 - m_2}$$

i. Substitute numerical values into the algebraic equation and arithmetically solve for the atomic weight of the other unknown.

$$X_1 = \frac{35.45 - 36.97}{34.97 - 36.97} = \frac{-1.52}{-2.00} = 0.760 = 76.0\%$$

note,  $X_2 = 1 - 0.76 = 0.24$ , so it is 24.0% of the higher isotope