Worksheet 3B Solutions

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The probability of finding a 1-D quantum mechanical particle in a range from x and $x + dx$ is given by:

$$\int_{x}^{x+dx} \Psi_n * (x)\Psi_n(x)dx \tag{1}$$

The eigenfunctions obtained for the particle in a box is

$$\Psi_n(x) = B \sin\Big(\frac{n\pi x}{a}\Big) \tag{2}$$

where n is the quantum number, a is the length of the box, and B is a constant.

1. (a) What is the most simplified expression for the probability density for particle in a box?
   The *probability density* is given by:

$$P(x) = \Psi_n * (x)\Psi_n(x) = B^2 \sin^2(\frac{n\pi x}{a}) \tag{3}$$

   (b) What is the general expression for the probability of finding the particle in a box of length a for any quantum level n?
   The probability of finding a particle in a box between b and c is given by:

$$P(b:c) = \int_{c}^{b} \Psi_n * (x)\Psi_n(x) = \int_{c}^{b} B^2 \sin^2(\frac{n\pi x}{a}) \tag{4}$$

   (c) Intuitively, if the particle is in the box, what is the probability of finding the particle in that box?
   The probability is 1.

2. The general solution for the integral of $sin^2()$ over all space is:

$$\int \sin^2(kz)dz = \frac{1}{2}z - \frac{1}{4k}\sin(2kz) \tag{5}$$

   (a) Find the numeric value for the probability that the particle is in the box using the above expression.
   For our case, $k = \frac{n\pi}{a}$, so we have:

$$P(inbox) = \int_{0}^{a} B^2 \sin^2(\frac{n\pi x}{a}) = B^2\Big(\frac{1}{2}x\Big|_0^a - \frac{a}{4n\pi}\sin\Big(\frac{2n\pi z}{a}\Big)\Big|_0^a\Big) = B^2\frac{a}{2} \tag{6}$$

   Without a specific value for the normalization constant, we cannot obtain a numerical value. However, the value should be 1.

3. (a) For equation (6) to be normalized, we must have $B = \sqrt{\frac{2}{a}}$

(b) What is the normalized eigenfunction for particle in a box?

The normalized eigenfunction is:

$$\Psi_n(x) = B \sin\left(\frac{n\pi x}{a}\right) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \tag{7}$$

(c) Using the normalized eigenfunction, what is the probability of finding the particle in the box, between 0 and a?

As before we have:

$$P(inbox) = \int_0^a B^2 \sin^2(\frac{n\pi x}{a}) = B^2 \left(\frac{1}{2}x\Big|_0^a - \frac{a}{4n\pi} \sin\left(\frac{2n\pi z}{a}\right)\Big|_0^a\right) = \frac{2}{a}\frac{a}{2} = 1 \tag{8}$$

which makes physical sense.