

Worksheet 2 Solutions

Jonathan Sarker

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1. Answer the following questions for this function of t and x :

$$f(x, t) = \exp(-3t) \cos(2x) \quad (1)$$

- (a) Which part(s) of $f(x, t)$ vary with x ?
Only $\cos(x)$ varies with x .
- (b) Which part(s) of $f(x, t)$ are constant when x is varied?
 $\exp(-3t)$
- (c) What is $\frac{\partial f}{\partial x}$?
 $\frac{\partial f}{\partial x} = -2 \exp(-3t) \sin(2x)$
- (d) If of $f(x, t) = a(x)b(t)$, using the definition of $f(x, t)$ above, what is $a(x)$?
 $a(x) = \cos(2x)$
- (e) What is $b(t)$?
 $b(t) = \exp(-3t)$

2. For any function defined as $u(x, t) = X(x)T(t)$:

- (a) Write the general expression for $\frac{\partial u}{\partial x}$ in terms of $u(x, t) = X(x)T(t)$
 $\frac{\partial u}{\partial x} = T(t) \frac{dX(x)}{dx}$
- (b) Similarly, how would you write $\frac{\partial u}{\partial t}$?
 $\frac{\partial u}{\partial t} = X(x) \frac{dT(t)}{dt}$

3. The equation for the vibration of a string is:

$$\frac{\partial^2 u(x, t)}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 u(x, t)}{\partial x^2} \quad (2)$$

where x is the distance along the string and t is time.

4. Given that $u(x, t) = X(x)T(t)$, rewrite the equation so that one side depends only on x and the other depends only on t :

$$\frac{1}{T(t)} \frac{\partial^2 T(t)}{\partial t^2} = \frac{1}{v^2} \frac{1}{X(x)} \frac{\partial^2 X(x)}{\partial x^2} = K$$

5. What about $u(x, t)$ makes separation of variables possible?

$u(x, t)$ is a product of two different functions, of the form $X(x)T(t)$ which allows for separation.

6. Is $u(x, t) = x \exp(-3t) \cos(2x)$ separable? What about $u(x, t) = \exp(-ix) \cos\left(\frac{2x}{\pi t}\right)$?

The first is separable because it is of the form $X(x)T(t)$. The second is not because the $\cos()$ cannot be factored into the form $X(x)T(t)$.