Worksheet 1

Gaussian distribution: $D(x) = N_0 e^{-N_1 x^2}$

The integrated value A:

$$A = \int_{-\infty}^{\infty} N_0 e^{-N_1 x^2} dx = \left\{ \int_0^{\infty} e^{-ax^2} dx = \frac{1}{2}\left(\frac{\pi}{a}\right)^{1/2} \right\} = N_o \cdot 2\int_0^{\infty} e^{-N_1 x^2} dx = N_o \cdot 2 \cdot \frac{1}{2}\sqrt{\frac{\pi}{N_1}} = N_0\sqrt{\frac{\pi}{N_1}}$$

The mean <x>:

$$x = \int_{-\infty}^{\infty} x \cdot N_0 e^{-N_1 x^2} dx = \left\{ \int_0^{\infty} x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}, n = 0 \right\} = N_o \cdot \left(\int_{-\infty}^{0} x \cdot e^{-N_1 x^2} dx + \int_0^{\infty} x \cdot e^{-N_1 x^2} dx \right) = N_0 \cdot \left(\left(-\frac{0!}{2a^{0+1}}\right) + \frac{0!}{2a^{0+1}} \right) = 0$$

The most probable value x_{mp}:

$$\frac{d(D(x))}{dx} = N_0 \cdot (-2N_1) \cdot x \cdot e^{-N_1 x^2} = 0$$

$x_{mp}=0$

The standard deviation σ_x:

$$\sigma_x = \int_{-\infty}^{\infty} (x - \bar{x})^2 \cdot N_0 e^{-N_1 x^2} dx = N_o \cdot \int_{-\infty}^{\infty} x^2 \cdot N_0 e^{-N_1 x^2} dx = \left\{ \int_0^{\infty} x^{2n} e^{-ax^2} dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n}\sqrt{\frac{\pi}{a}}, n = 1 \right\} = N_o \cdot 2\int_0^{\infty} x^2 \cdot N_0 e^{-N_1 x^2} dx = N_0 \cdot 2 \cdot \frac{1}{2^{1+1} N_1^{\ n}}\sqrt{\frac{\pi}{N_1}} = \frac{N_0}{2\ N_1^{\ n}}\sqrt{\frac{\pi}{N_1}}$$

The expression for finding x exactly at 0 for a *Gaussian probability distribution is*

$$D(0) = N_0 e^{-N_1 x^2} = N_0 e^{-N_1 0^2} = N_0$$

The constants N_o and N_1 in the Gaussian distribution are expressed in terms of each other:

$$A = \int_{-\infty}^{\infty} N_0 e^{-N_1 x^2} dx = N_o \cdot 2\int_0^{\infty} e^{-N_1 x^2} dx = N_o \cdot 2 \cdot \frac{1}{2}\sqrt{\frac{\pi}{N_1}} = N_0\sqrt{\frac{\pi}{N_1}} = 1$$

$$N_0 = \sqrt{\frac{N_1}{\pi}}$$

The expression for finding x between and 0 and +∞ for a *Gaussian probability distribution* when N_o and N_1 are expressed in terms of each other:

$$N_0 \int_0^{\infty} e^{-N_1 x^2} dx = N_o \cdot \frac{1}{2}\sqrt{\frac{\pi}{N_1}} = \sqrt{\frac{N_1}{\pi}} \cdot \frac{1}{2}\sqrt{\frac{\pi}{N_1}} = \frac{1}{2}$$